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# Mixed traffic driver behavioral modeling at urban merge section: an experimental study

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# ABSTRACT

In this study, the relation between macroscopic parameters (speed, flow, density, occupancy) and microscopic parameters (lateral clearance, average gap, space headway, lateral movement duration, overtaking time, overtaking distance, relative speed) is evaluated by considering vehicle-type-dependent factor using the trajectory data collected at a five-lane urban merge and non-merge sections in Malaysia. Traffic data extractor software is utilized to extract the flow, speed, and trajectory data of the vehicles at both the merge and non-merge sections of urban highway. Both SPSS software and MATLAB software were used to conduct statistical and numerical modeling of the extracted field data. The results show a significant variation in vehicle-type dependant driving behaviors at merge and non-merge section. These parameters are useful in the development of simulation models for the merging part on a multi-lane urban highway. The findings are also useful for estimating the potential collisions under mixed traffic conditions in road safety management.

#### **KEYWORDS**

Mixed traffic; vehicle-type; driving behavior; dynamic parameters; merge sections; lateral movement; overtaking behavior

# Introduction

Drivers' tactical maneuvering decisions under different traffic states form the basis of driver behavioral modeling. The critical transportation studies like safety analysis and capacity analysis utilize aggregate traffic flow characteristics from these models which are deduced from individual driver behavior. A vehicle's attitude and motion depend upon its stability control, which is a significant issue while driving, especially in mixed traffic conditions. Directional control is the result of how accurate the driver interprets the vehicle motion, which in turn affects the vehicle stability. Lateral and longitudinal monitoring of the vehicle is the implication of directional control. The fluctuations in lateral and longitudinal controls of vehicle amplify at unsignalized merging sections under mixed traffic conditions, due to variability in vehicles' static and dynamic characteristics. Hence, at these scenarios, drivers need to control the car in longitudinal and lateral directions simultaneously giving rise to two-dimensional (2D) driving behavior. This 2D behavior gives rise to the inter-dependency of both lateral and longitudinal parameters. Hence, a comprehensive study of this 2D driving behavior, especially at urban merge sections under mixed traffic conditions, is necessary.

The capacity of a road depends upon the macroscopic traffic characteristics (i.e., speed, flow, density) which in turn depend upon the dynamic parameters (e.g., speed and acceleration) of individual vehicles. The relation between the individual vehicle's dynamic parameters such as speed and lateral/longitudinal acceleration determines the drivers' erratic behavior, especially at merge sections under mixed traffic stream. The lateral movement behavior is an essential parameter for representing vehicle interactions at intersections for the development of simulation models for lateral stability and safety analysis. In this study, we evaluate drivers' movements based on their vehicle types at an urban merge section under mixed traffic conditions and compare them to a non-merge section. The dynamic parameters like speed, deceleration rates, space headway, lateral clearance, average gaps, overtaking action of different vehicle types are evaluated with macroscopic parameters (i.e., flow, density, occupancy). The abreast driving behavior of vehicles at merging areas leads to the serpentine motion of vehicles through lateral and longitudinal acceleration/deceleration as compared to that of a non-merge section, giving rise to 2D driving behavior. Hence, the lateral and longitudinal behavioral study has become a critical issue at these merging areas under mixed traffic scenario.

# **Review of earlier studies**

The traffic flow evaluation and its consequent effect on the capacity and safety at the location of merging require a study on traffic flow variations, individual vehicle speed and time headway at the merging section (Kumar, Arkatkar, and Joshi 2019). Past studies that took place to evaluate the driver behavior at merging sections were mostly under homogeneous traffic conditions (Elefteriadou, Roess, and McShane 1995; Kerner and Rehborn 1997; Ng 2021). Roy and Saha (Roy and Saha 2018) developed headway distribution model for different vehicle types under mixed traffic conditions for capacity and safety evaluation of urban traffic flow. However, their work is limited to two-lane roads, further exploration is needed to understand and estimate the headway distributions of vehicles on multilane roads and intersections. Loulizi et al. (Loulizi, Bichiou, and Rakha 2019) in their work implemented naturalistic driving data to analyze time-gaps between vehicles under car-following scenario. They observed significant variations in time-gaps between drivers with speed variations above 54 km/h. However, their study was

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conducted under homogeneous traffic conditions where there is no influence of vehicle type characteristics on traffic flow. Asaithambi et al. (Asaithambi et al. 2016) in their work evaluated different traffic flow characteristics using different car-following models under heterogeneous traffic conditions. Further investigation is required to evaluate the impact of different traffic flow parameters on different vehicle combinations at different urban road sections. An attempt has been made by Luo et al. (Luo, Li, and Wang 2010) to correlate the variations in microscopic traffic parameters to macroscopic characteristics under transitional state of traffic by implementing time headway. Their work did not consider the impact of multiple lanes and different vehicle types on the transitional traffic flow state.

Maurya et al. (Maurya et al. 2016) and Moinuddin et al. (Moinuddin et al. 2017) in their works attempted to evaluate the speed, flow and headway distributions of vehicles for different traffic density levels under mixed traffic conditions. They concluded that different vehicle pairs show varied speed and headway distribution patterns at different density levels. However, these distribution patterns were for a straight road section without any intersections since the presence of unsignalized intersections have a profound effect on the traffic flow. The impact of heavy vehicles upon the merging section capacity was analyzed and concluded that with an increase in 1% of heavy vehicles, there would be a 1% decrease in capacity (Sarvi and Kuwahara 2007). Merging sections are one of the major causes of breakdowns, capacity drops and crashes on urban roads (Weng, Xue, and Yan 2016). A relationship is established between the speed of vehicles and their lateral and longitudinal acceleration/decelerations on different road sections for different vehicle types and speed prediction models (Mahapatra and Maurya 2018). A good number of speed and acceleration/ deceleration based models are built for various applications in the past research. Majority of the studies on longitudinal vehicle behavior were conducted on signalized intersections (Akçelik and Besley 2001)(Wang et al. 2004). A minimal number of studies on longitudinal behavior of vehicles in mixed traffic situations exist in which they concluded that the longitudinal accelerations decrease with increase in the speed of vehicles (Bokare and Maurya 2016) (Maurya and Bokare 2012).

Though a good number of studies were conducted on the longitudinal interaction of vehicles in the traffic stream, the lateral interaction of vehicles needs more exploration. There is a need for further research in analyzing the interactions between vehicles under mixed traffic stream especially between two-wheelers and other vehicle types (Matcha et al. 2020). Mahapatra et al. (Mahapatra, Maurya, and Minhans 2016) evaluated the lateral movements of different vehicle types on straight road sections in three different cities. They derived relationships between lateral acceleration and longitudinal speed of different vehicles. However, there is a need for evaluation of lateral movement behavior of vehicles at different road sections to understand the variations in their lateral behavior which was not considered in their study. Mahapatra et al. (Mahapatra and Maurya 2015) studied the impact of different lane positions on the lateral movement behavior of vehicles by evaluating parameters such as lateral clearance and mean speed under mixed traffic conditions. They observed significant speed and headway variations between different lanes. However, no comparisons in variations in speed, headway distributions and lateral movements behavior between vehicle types were drawn on different road sections. The speed or trajectory of vehicles is determined by their lateral acceleration comfort and safety margin (Reymond et al. 2001).

From the past literature, the lateral acceleration increased up to  $4 \text{ m/s}^2$  for most of the drivers. In contrast, a lateral acceleration of

6–8 m/s<sup>2</sup> is hard for the drivers (Xu et al. 2015). The lateral movement behavior is one of the vital traffic flow characteristics that determines the behavior of drivers under heterogeneous traffic stream (Kanagaraj et al. 2015). Asaithambi et al. (Asaithambi and Joseph 2018) studied the duration of lateral shifts of different vehicle types under mixed traffic conditions. They observed a lateral shift duration ranging between 0.5–15 s for different vehicle types. However, they did not consider the impact of road geometrics such as intersections on the lateral shift behavior of different vehicles. A relationship between lateral headway and timeto-collision (TTC) was established in an experimental study by Bhargav and Chhabra (Bhargav Naidu and Chhabra 2018), where they observed that drivers with larger lateral headway have less TTC and vice versa, giving rise to oblique and no-lane-based carfollowing behavior.

A thorough evaluation of overtaking maneuvers was done by Hegeman et al. (Hegeman, Hoogendoorn, and Brookhuis 2005) on a two-lane rural highway by introducing the instrumented vehicle method to understand the driver behavior after an overtaking maneuver. The differences in overtaking durations with different speeds of vehicles were observed. An overtaking behavioral model was developed by Tang et al. (Tang et al. 2007) by considering factors like relative delay time for acceleration, lane-changing, deceleration, passing time, car-following safe-distance and spacetime evolution of vehicle movement. Llorca and Garcia (Llorca and García 2011) developed a new method to understand passing maneuvers on highways by installing video cameras at fixed positions to analyze the data from 234 maneuvers on four passing zones and to evaluate the speed difference between passing and impending vehicles. Chandra et al. (Chandra and Shukla 2012) in their work estimated the impact of acceleration rates of different vehicle types on their overtaking behavior. In their study they observed an inverse relation between the acceleration rate and overtaking speed. However, this study is limited to a divided straight highway section and an in-depth exploration is required in estimating and correlating various parameters influencing overtaking characteristics of vehicle types at different road sections.

Ghods (Ghods 2013) developed an overtaking gap-acceptance model based on the driver's perception of TTC to the opposing vehicle in order to determine the critical TTC through model calibration and validation of observed data. The factors affecting the speed of overtaking vehicle were determined in Malaysia by a video-graphic survey on a stretch of 280 m road during peak and off-peak hours. The factors like driver's perception time, overtaken vehicle speed, acceleration of the passing vehicle, passing time and safety limit determined the overtaking behavior of vehicles (Hassan et al. 2014). However, a large amount of data from different road sections needs to be evaluated to produce significant and reliable relationship between all the parameters influencing the overtaking behavior. Asaithambi et al. (Asaithambi and Shravani 2017) evaluated the behavior of overtaking maneuver of different vehicle types under mixed traffic conditions on an undivided highway. However, they did not evaluate the variations in overtaking maneuvers of different vehicle types as a function of macroscopic parameters such as traffic volume and density, since these macroscopic parameters have a profound impact on drivers overtaking characteristics. There exists a void in past research in estimating the driving behaviors (passing maneuvers, lateral shifts, etc.) that are unique to mixed traffic flow due to limited availability of trajectory data for estimation of these parameters (Matcha et al. 2020; Asaithambi, Kanagaraj, and Toledo 2016).

Although there exists a good number of studies on drivers behavioral modeling under mixed traffic conditions, there still exists a void in the past research. i) Very few existing studies are related to merging maneuver of different vehicle types and their influence on microscopic parameters. ii) Most of the existing behavioral studies did not thoroughly explore the relationship between microscopic and macroscopic traffic flow parameters at merging sections of urban highways consisting of mixed traffic flow. iii) Most of the existing studies on mixed traffic flow were conducted on a single road section and the parameters influencing driver's behavior were not compared to other road types for a better understanding of their behavioral movements. iv) There is a need for exploring and comparing different macroscopic and microscopic parameters through a good trajectory data set of different road sections based on vehicle type factor for a thorough understanding of mixed traffic vehicle movements.

In order to address the above-mentioned issues and to evaluate the driver's behavioral movements of various vehicle-types in a more accurate manner, this study attempts to model and explore the relationship between microscopic and macroscopic traffic flow parameters at merging sections by implementing vehicle-type dependant factor. The motivations and contributions of this study are:

- (1) This study evaluates the effect of merging on behavioral movements of different vehicle types (i.e., lateral, longitudinal, and overtaking behavior) at an urban merge section under mixed traffic conditions.
- (2) This study thoroughly explores the relationship between microscopic and macroscopic traffic flow parameters at

merging section of urban highways consisting of mixed traffic flow.

- (3) This study implements vehicle-type dependent factor to evaluate individual vehicle movements and their behavior under varying traffic flow conditions at merge and nonmerge sections under mixed traffic flow conditions.
- (4) This study further compares the vehicle type dependant traffic flow parameters and their variations on drivers' behavioral movements at merge and non-merge sections of urban highway under mixed traffic conditions.

The rest of the paper is organized in the following manner. Section 3 presents the methodology flowchart of this study. Section 4 presents data collection and extraction. Section 5 presents the analyzed traffic flow parameters (longitudinal behavior, lateral behavior, overtaking behavior) of different vehicle types and their comparison at merge and non-merge sections. Finally, Section 6 presents the summary and conclusion part of the study. The overall study methodology is presented in Figure 1.

# **Data acquisition**

Field data was acquired from a five-lane divided high-speed urban merge section in Selangor, Malaysia. Field reconnaissance survey was conducted along the Persiaran Kewajipan highway of 10 km in Subang Jaya. The study merge section was chosen based on





Figure 2. Overall view of study merge section.

availability of vantage point for camera recording to acquire traffic flow data, inspected during reconnaissance survey. The overall detailed study area (merge and non-merge sections) is shown in Figure 2. The study area is a five-lane divided urban road having a carriageway width of 17.5 m ( $3.5 \text{ m} \times \text{five lanes}$ ) on each side.

From the preliminary survey it has been observed that the peak hour traffic in this merging location is from 0800 to 1000 hours and 1600 to 1800 hours. Also, there is a spike in the flow rate at 1200 hour which is not regarded as peak hour as the flow rate is not high enough. The data is collected for approximately one hour during the evening peak hour. The data is collected through video recording performed at the vantage point shown in Figure 2 during the peak hours mentioned above. The one-hour free flow data has been collected during off-peak hours to determine free flow traffic state. The travel times of different vehicles from the respective entry points to the exit points of the merging section are extracted. Traffic volume, speed, and occupancy values are extracted for every one minute by using the Traffic Data Extractor software. Other parameters such as vehicle composition, lanechanging or merging and diverging movements of vehicles during the peak and off-peak hours are extracted as well. The merging section captured and analyzed is shown in Figure 2.

# **Data extraction**

The data extracted through video recording was analyzed using the traffic data extraction software (Munigety, Vicraman, and Mathew 2014) to study the flow, speed, and trajectory data of the vehicles at the merge section. The path traversed by a particular vehicle over a period of time denotes vehicle trajectory. Vehicle trajectory data forms an important part of microscopic traffic data through which various microscopic details such as lane-change durations, headways, etc., can be computed. The framework of traffic data collection and extraction is shown in Figure 3.

- First, the study merge section is videotaped from a preferable vantage point (in this study a footbridge), and the distance between the reference points on the field are calculated.
- Second, the traffic flow recording is uploaded unto Traffic Data Extractor (TDE) software and an update interval of

 $0.5s (\leq 1 \text{ s})$  was set to extract trajectory data, such that individual vehicles' behavioral movements (stop, turn, slow down, speed up, lane change movements, etc.) and many other measures of effectiveness (MOEs) are captured (Authors 2015; RAJU et al. 2017). This update interval establishes the frame rate for the extraction of trajectories.

- Third, the calibration rectangle or trap length and width are identified in correlation with the reference points in the video. The reference points A, B, C, and D were clicked in order, and based on the reference points the dimensions of the rectangular trap are determined as shown in Figure 2. In this study, the length and breadth of rectangular trap are  $50 m \ge 15 m$  at merge section and  $200 m \ge 15 m$  at non-merge section respectively.
- Fourth, the trajectory of a particular vehicle is extracted by selecting the type of vehicle being tracked and by clicking on its front left or front-right tire till it disappears from the video frame, and the video automatically updates to its next frame. The process continues until the required number of vehicle trajectories are extracted.
- Finally, the trajectory data are exported to get the output in . csv format. The traffic volume for the 1-minute interval was analyzed by manually counting the vehicles passing through the section. The timestamp of a vehicle at the upstream and downstream of the section are recorded to determine the time and space headway for further analysis. The selection of duration took place to record maximum variation in traffic flow based on visual inspection.

# Trajectory data smoothening

TDE software is a semi-automated extraction tool where the vehicles are tracked and extracted manually which gives rise to human errors. This results in high instant velocities and acceleration rates causing inconsistency in obtained field data (RAJU et al. 2017). To address this issue Moving Average Method of smoothening technique (Roland 2004) is implemented to smoothen the extracted trajectory data by means of a MATLAB code. Since, moving average method is found suitable in improving the consistency in vehicles trajectories with simple effort (RAJU et al. 2017). Mathematically,



Figure 3. Data collection, extraction process, and camera calibration in TDE software.



Figure 4. Space-time plots of vehicles a) merge section, b) non-merge section.

a moving average is a kind of complexity where the mean is calculated by extraction of equal amount of data on both sides of a central value. This establishes alignment of the mean value with the variations in the extracted data. Therefore, the central value is estimated by computing the mean value by utilizing the data that is equally spaced on either side of the point in the series. This method utilizes odd number of data points to estimate the central value.

The present study implemented moving average of 3-points, 5-points, and 7-points to smoothen the trajectory data, where these selected subsets of points were considered, and central value

is calculated. As per the above methodology the trajectory data was smoothened and compared with the extracted trajectory data for internal consistency.

From the extracted trajectory data space-time curves were plotted for vehicles at both merge and non-merge sections. Figure 4 (a) and (b) shows the space-time plots of vehicles at merge section for a period of 10 minutes and at non-merge section for a period of 15 minutes.

Vehicle type	PCU value
Passenger car	1.0
Light commercial vehicle (LCV)	1.19
Motorcycle	0.25
Heavy truck	2.27
Passenger bus	2.08

# **Traffic flow observations**

# Traffic composition

In order to normalize the heterogeneity of traffic conditions, the vehicles are converted to Passenger Car Units (PCU) to determine traffic volume and density in the present study. The PCU values are adopted from Malaysian Highway Capacity Manual (MHCM) (Vien, Wan Ibrahim, and Mohd 2006). The PCU values for different vehicle types are referred from MHCM (2006) as shown in Table 1. The study section comprised of cars (62%), motorcycles (28%), and heavy vehicles (buses and trucks) (10%) of the traffic. The static and dynamic features different vehicle types analyzed from the extracted data are shown in Table 2.

The variations in speeds of different vehicle types as a function of traffic volume at merge and non-merge sections are shown Figure 5 (a), (b), and (c).

Table 2. Traffic characteristics at merge and non-merge sections.

		Merge sect	on				
Vehicle type	Sample size	Speed (m/s) $\mu_v$ ( $\sigma_v$ )	Acceleration (m/s <sup>2</sup> ) $\mu_a$ ( $\sigma_a$ )	Sample size	Speed (m/s) $\mu_v$ ( $\sigma_v$ )	Acceleration (m/s <sup>2</sup> ) $\mu_a$ ( $\sigma_a$ )	Width (m)
Car	3234 (62%)	11.11 (1.93)	1.67 (1.56)	3754 (59.5%)	14.16 (1.77)	1.48 (1.73)	1.50-2.50
Heavy vehicle	1656 (28%)	9.78 (1.28)	1.43 (0.61)	1479 (23.4%)	10.84 (1.17)	1.31 (0.84)	2.50-3.00
Motorcycle	986 (10%)	13.30 (2.67)	2.59 (1.85)	1068 (16.9%)	15.47 (2.47)	2.17 (1.62)	0.50-0.75

 $\mu$  is mean value,  $\sigma$  is standard deviation.





(b)



(c)

Figure 5. Vehicle speed versus traffic volume (a) car, (b) heavy vehicle, (c) motorcycle.

Table 3. T-test results for variation in vehicles speed at merge and non-merge section.

	Merge section	Non-merge section		
Vehicle type	Mean speed (km/hr)	Mean speed (km/hr)	Speed reduction (%)	p-value
Car	41.51	59.51	30.2	0.0168
Heavy vehicle	31.98	46.36	31.0	0.0188
Motorcycle	45.72	59.47	23.1	0.0400

A significant reduction in the speeds of different vehicles is observed at the merge section compared to non-merge section on the five-lane urban road. To prove the significance a statistical t-test was conducted, and the results are shown in Table 3. From the t-test results it is clear that there exists a significant variation in speeds of cars (p - value = 0.0168 < 0.05), heavy vehicles (p - value = 0.0188 < 0.05). and motorcycles (p - value = 0.040 < 0.05) at merge section compared to non-merge section.



Figure 6. Cumulative frequency distribution curves for cars at merge and non-merge sections.



Figure 7. Cumulative frequency distribution curves for heavy vehicles at merge and non-merge sections.

The desired speed being 90 km/hr and heavy vehicle speeds are much affected by the increase in traffic flow. However, twowheelers showed comparatively a slow pace in speed reduction, suggesting a vast number of within-lane movements and filtering through the traffic phases.

# Speed distribution of different vehicle types at merge and non-merge section

The speed distributions of different vehicle types at both merge and non-merge sections are determined and compared by analyzing the field data and plotting cumulative distribution curves. The cumulative speed curves for different vehicle types at merge and nonmerge sections are shown in Figures 6, 7 and 8 respectively.

There exist significant variations in speed distributions of different vehicles at merge and non-merge sections. The speed distributions of vehicle types at merge section are considerably lower as compared to non-merge section, since two different traffic streams having varied design speeds would lead to increase in lanechanging rates and reduction in speeds.

#### Longitudinal movement behavior

#### Identification of leader-follower pairs

From the extracted trajectory data different types of following behavior are observed namely car following and staggered following. In car-following behavior, the follower vehicle strictly aligns with the leader vehicle. In case of staggered-following behavior, the lag vehicle often gets staggered with the front vehicle giving rise to partial following behavior due to lane-less traffic movements observed at merge section.

From the extracted trajectory data, the amount of overlap (Kanagaraj et al. 2015) between the front and lag vehicles was observed and estimated to identify following behavior. For a given subject vehicle a leader to the subject vehicle is identified. Any type of front vehicle which laterally overlaps with the follower vehicle within the study merge and non-merge section is identified as a leader vehicle. The lateral and longitudinal coordinates of front center of each vehicle ( $x_{ci}$ ,  $x_{ci}$ ) are extracted from the trajectory data to define which vehicle follows the other. The left and right lateral bound coordinates of each

vehicle are computed per time instant t as shown in Equations (1) and (2).

$$x_{l_i}(t) = x_{c_i}(t) - \frac{w_i}{2} - s_i(t)$$
(1)

$$x_{r_i}(t) = x_{c_i}(t) + \frac{w_i}{2} + s_i(t)$$
(2)

Here i : 0, 1, 2, n vehicle index  $x_{c_i}$  denotes lateral coordinate of front center of vehicle i,  $x_{l_i}$  denotes lateral coordinate of front left bound of vehicle i,  $x_{r_i}$  denotes lateral coordinate of front right bound of vehicle i,  $w_i$  denotes width of vehicle i,  $s_i$  denotes safe lateral distance of vehicle i.

The amount of overlap between different leader-follower pairs was estimated from the trajectory data using the MATLAB code. For a given leader-follower pair, the lateral overlap may exceed 100% if the follower vehicle type is narrower than the leader vehicle. If the overlap between the front and lag vehicles less than half the width of the lag vehicle width, then it indicates staggered following. Hence, in order to define leader-follower pairs, the longitudinal position of the front vehicle should be at a distance L that could influence the movement of follower vehicle as shown in Equation (3) (Papathanasopoulou 2018)

$$y_{follower}(t) \le y_{leader}(t) \le y_{follower}(t) + L$$
 (3)

Also, a part of the front side of one vehicle should overlap with front side of other vehicle as shown in Equations (4) and (5)

$$x_{l_{follower}}(t) \le x_{r_{leader}}(t) \tag{4}$$

$$x_{l_{leader}}(t) \le x_{r_{follower}}(t) \tag{5}$$

Where,  $y_{follower}$  denotes longitudinal coordinate of follower vehicle,  $y_{leader}$  denotes longitudinal coordinate of leader vehicle,  $x_{l_{follower}}$  denotes lateral coordinate of front left bound of follower vehicle,  $x_{r_{leader}}$  denotes lateral coordinate of front right bound of leader vehicle,  $x_{l_{loader}}$  denotes lateral coordinate of front left bound of leader vehicle, and  $x_{r_{follower}}$  denotes lateral coordinate of front left bound of leader vehicle, and  $x_{r_{follower}}$  denotes lateral coordinate of front right bound of leader vehicle.

If these conditions are satisfied at the same instant of time, then each vehicle pair is considered as leader-follower pair.



Figure 8. Cumulative frequency distribution curves for motorcycles at merge and non-merge sections.

Estimation of space-headway for different leader-follower pairs

Space headway can be defined as the distance between same points of two consecutive vehicles following each other. The dynamic parameters such as speeds of different vehicles varied as high as 15 m/s to as low as 9 m/s, showing varied maneuvering capabilities of each vehicle type. Also, due to their wide range of difference in static characteristics like the size, they exhibit their own acceleration capabilities. Hence, in order to determine the longitudinal movements of different vehicles, the space headways of different vehicle pairs are shown in Table 4 and Figure 9 (a), (b), and (c). From the analyzed data, it can be suggested that the vehicle-following nature exists, since the mean and standard deviation of vehicle pairs are quite close. However, the mean space headway varied for different vehicle pairs. The following mean headway in case of motorcycles are comparatively less than other vehicle types, showing high manoeuvrability due to their small size. Hence, these results show the presence of the following behavior but varies for different vehicle pairs.

The variation in space headway between different leader follower pairs is also analyzed at different levels of congestion, i.e., volume-tocapacity ratio (V/C) at both the merge and non-merge sections. The variation in space headway as a function of (V/C) ratio at merge and non-merge sections is shown in Figures 10 and Figures 11.

- From Figures 10 and 11, it can be evaluated that the space headway between different vehicle pairs at merge section is lower compared to that of the non-merge section for a given level of congestion or (V/C) ratio.
- This reduction in space headway is due to the merging of two different traffic streams having different design speeds giving rise to vigorous lane changes and speed reduction, which is not the case in non-merge section.
- In both merge and non-merge sections motorcycle combination pairs maintained the least space headway due to their smaller dimensions and higher maneuverable capabilities, followed by car and truck combination pairs.

# Estimation of reaction time and maximum deceleration rate for different vehicle pairs

Safe-distance based longitudinal models are better suited to estimate the behavioral movements of various vehicle types under mixed traffic conditions (Matcha et al. 2020). Krauss (Krauß 1998) developed a modified safe-distance based longitudinal model which is found to be good fit for mixed traffic flow (Ravishankar and Mathew 2011). The proposed model forecasts

Table 4. Space headway maintained by different vehicle types.

L-F	Total observations	$\begin{array}{l} \text{Leader vehicle} \\ \text{mean velocity } {\mu_v}^L \\ ({\sigma_v}^L) \end{array}$	Follower vehicle mean velocity $(\mu_v^F)$ $(\sigma_v^F)$	Mean space- headway (µv <sup>LF</sup> ) (σv <sup>LF</sup> )
C-C	567	15.14 (2.94)	13.41 (2.67)	10.11 (5.21)
C-H	325	10.89 (1.07)	9.47 (0.84)	13.57 (6.01)
C-M	212	13.04 (1.54)	12.56 (1.88)	11.68 (2.21)
H-C	302	11.02 (1.73)	10.38 (1.65)	12.69 (5.03)
H-H	113	11.12 (2.39)	10.37 (2.87)	14.58 (6.18)
H-M	109	10.09 (1.19)	9.82 (1.05)	11.05 (2.41)
M-C	278	14.01 (3.12)	12.82 (3.19)	5.09 (3.08)
M-H	134	11.31 (1.02)	11.19 (0.93)	6.97 (5.06)
M-M	376	13.54 (2.17)	13.09 (2.11)	3.22 (2.03)

L – Leader, F – Follower, C – Car, H – Heavy vehicle, M – Motorcycle,  $\mu_v^{L-}$  Mean velocity of leader vehicle,  $\sigma_v^{L}$  – Standard deviation in mean velocity of leader vehicle,  $\mu_v^{F}$  – Mean velocity of follower vehicle,  $\sigma_v^{F-}$  Standard deviation in mean velocity of follower Vehicle,  $\mu_v^{LF}$  – Mean space-headway for leader-follower pair,  $\sigma_v^{LF-}$  Standard deviation in mean space-headway for leader-follower pair.

the velocity of the follower vehicle j, based on its longitudinal gap with the leading vehicle i in the preceding time step as shown in Equation 1.

$$v_t^j = -\tau.b + \sqrt{b^2.\tau^2 + (v_{t-1}^i)^2 + 2.b.g_{t-1}}$$
 (6)

Here, the velocity of the following vehicle is denoted by, *b* denotes the maximum deceleration rate in  $m/s^2$ ,  $\tau$  denotes the reaction time in *s*, the velocity of leader vehicle is denoted by  $v_{t-1}^i$  at t-1 time step in m/s, and the gap between the leader-follower vehicle pair is denoted by  $g_{t-1}$ .

Implementing the vehicle-type dependent factor in equation 1 of the above proposed safe-distance model, the speed of following vehicle type j with leading vehicle type i is formulated as shown below:

$$v_t^j = -\tau^{ij} \cdot b^{ij} + \sqrt{(b^{ij} \cdot \tau^{ij})^2 + (v_{t-1}^i)^2 + 2 \cdot b^{ij} \cdot g_{t-1}}$$
(7)

In homogeneous traffic conditions, the reaction time and maximum deceleration parameters of all the vehicles remain almost the same due to the presence of mostly similar vehicle types. However, these parameters vary depending upon the vehicle pair in case of heterogeneous traffic conditions as shown in equation 2. The reaction time and maximum deceleration rate are calibrated for each vehicle pair using the modified Krauss model.

A total of 185 datasets extracted from vehicle trajectory data were considered for evaluation. The reaction time and maximum deceleration rate for different leader follower pairs was evaluated through video analysis under free flow traffic conditions such that vehicles can accelerate and decelerate to their maximum extent. The deceleration rate for each follower vehicle in the pair is computed from the second-by-second speed data using the following equation.

$$d_{(t2)} = \frac{(\nu_1 - \nu_2)}{(t_2 - t_1)} \tag{8}$$

where  $d_{(t2)}$  denotes deceleration  $(m/s^2)$  at time  $t_2$ ,  $v_1$  and  $v_2$  denote speeds (m/s) of follower vehicle at time  $t_1$  and  $t_2$  (sec) respectively.

The process of deceleration starts from the moment where deceleration values computed from Equation (8) are greater than or equal to 0.1  $m/s^2$  continuously for the next 5 seconds. The vehicles speed becomes zero at the end of deceleration process. This algorithm is utilized to calculate deceleration values of different vehicle pairs.

Table 5 presents the reaction time and maximum deceleration rates for each vehicle pair observed at merge and non-merge sections.

- Firstly, the modified vehicle-type-dependent Krauss model evaluates the longitudinal movements in a better way compared to the traditional one. The prediction error in traditional Krauss car-following model is high compared to modified Krauss model.
- Huge variations can be found in the reaction time for different leader-follower pairs. Motorcycles have an average reaction time of 0.90 s, the lowest among the vehicle pairs. Moreover, it is also observed that slow-moving vehicles have a higher reaction time.
- In case of M-H pair, least deceleration rates were observed due to motorcycles that are exhibiting lower static and higher maneuvering capabilities as compared to heavy vehicles indicating swerving action instead of higher decelerations. Moreover, the deceleration rates were also lower in case of







(b)



Figure 9. Space headway of different vehicle pairs at merge section (a) cars (b) heavy vehicles (c) motorcycles.



Figure 10. Space headway vs V/C ratio for different leader-follower at merge section.



Figure 11. Space headway vs V/C ratio for different leader-follower at non-merge section.

C-H, H-C pairs due to their large static characteristics and higher space-headway maintenance.

• From the reported error values, it can be concluded that the modified vehicle-type dependant Krauss model forecasts the longitudinal behavior of different vehicle pairs better when compared to the traditional one.

# Lateral movement behavior

# Lateral movement duration

A lateral movement is the lateral shift of a vehicle along the road width to a distance equivalent to the vehicle's width (Munigety and Rao 2013). The merging location encounters many interactions between the vehicles in the lateral direction. Vehicle trajectory

	Ν	lerge section		Non-merge section			
Follower- leader pair	Reaction time T (s)	Maximum deceleration b (m/s <sup>2</sup> )	RMSE (%)	Reaction time T (s)	Maximum deceleration b (m/s <sup>2</sup> )	RMSE (%)	
C-C	1.89	2.33	0.35	2.14	2.78	0.22	
C-H	2.45	0.52	0.17	2.67	0.79	0.27	
C-M	1.10	2.51	0.28	1.43	2.82	0.13	
H-C	2.42	1.26	0.34	2.76	1.55	0.39	
H-H	2.29	2.67	0.42	2.58	2.72	0.21	
H-M	1.37	2.25	0.29	1.41	2.75	0.47	
M-C	0.90	2.60	0.48	1.04	2.77	0.35	
M-H	2.19	0.25	0.22	2.38	0.47	0.36	
Traditional Krauss	3.49	2.34	0.69	3.67	2.45	0.73	

Table 5. Calibration of leader-follower reaction time and deceleration characteristics for traditional and vehicle-type Krauss car-following models.

C-Car, H- Heavy vehicle, M- Motorcycle

data is used in the estimation of lateral shifts. The initiation time of a lateral movement is the moment when subject vehicle type starts to shift laterally, and the completion time is the time when the vehicle completes a lateral shift (equals to its width). The threshold values of lateral shift depend upon the vehicle type since the width of the vehicles are considered as threshold values. The mean threshold values for different vehicle types are 1.75 m (cars), 2.5 m (heavy vehicles), and 0.6 m (motorcycles) respectively.

The initiation of lateral shift by subject vehicle type depends upon the speed of leader vehicle type. From trajectory data analysis, the relative velocities between different vehicle pairs are evaluated within the merge section. The velocity difference between the front leader and subject vehicle is defined as the front relative velocity. The velocity difference between the target leader vehicle and the target follower vehicle is defined as the target relative velocity. The estimated relative velocities were maximum when the distance between the leader and subject vehicle type is within 30 m. The parameters such as vehicle type, longitudinal spacing, velocity, lateral shift direction, and target lateral gap are extracted from 1150 datasets estimated from trajectory data using MATLAB programming code.

From the extracted datasets, 61% were right lateral movements and 39% were left lateral movements. Cars executed more lateral shifts of 47%, followed by motorcycles 34.5% and heavy vehicles 12.5%. The mean lateral movement duration of different vehicle types is shown in Table 6. These durations vary as high as 15 s (heavy vehicles) to as low as 1 s (motorcycles) as shown in Figure 12 (a)-(f). From these observations, it can be suggested that different vehicle types have different lateral maneuvering capabilities because of variations in their static and dynamic characteristics.

The statistical significance of this difference is verified by conducting one-way ANOVA test assuming the null hypothesis that all vehicle types have the same lateral movement duration. The p-value of the result is less than 0.001 rejecting the null hypothesis with a 99% confidence level.

Table 6.	Mean	lateral	shift	duration	of	different	vehicle	types	at	merge	section.

Vehicle yype	TotaloObservations	Mean duration (s)
Car	542	5.85
Heavy vehicle	211	8.90
Motorcycle	397	3.01

- The direction of lateral movement influences the duration of lateral shift. From the estimated results, the mean right lateral movement duration is 2.5*s* which is higher than the left lateral duration 2.1*s*. This is due to the fact that the speed of traffic is faster in the right lanes in Malaysia.
- The right lane changing movements of vehicles are executed in a cautious manner to avoid conflicts with high-speed traffic in the right lanes.
- Heavy vehicles exhibited higher mean duration in their right lateral shift compared to other vehicle types due to their larger static characteristics and lower maneuverable capabilities.

Further investigation was carried out to evaluate the influence of leader vehicle type on lateral movement durations of subject vehicle. Table 7 presents the mean lateral movement durations for different leader-follower pairs.

- The mean lateral movement duration is minimum in the vehicle pairs with motorcycle as follower (1.5–3.0 s). This is due to the fact that motorcycles have lower static characteristics and high maneuverable capabilities.
- The car and heavy vehicle pairs where the leader vehicle is a heavy vehicle recorded high lateral movement durations (5.8 s) due to the greater static characteristics of heavy vehicle.
- All vehicle types preferred to make a right lateral shift if the leader vehicle is a heavy vehicle, because of their lower speeds and larger dimensions keeping them mostly to left slow moving lanes.
- The number of overtaking for all vehicle types are higher if the leader vehicle is a heavy vehicle.

The variations in lateral movement durations of different vehicle types are also analyzed at different levels of congestion i.e., volume-to-capacity ratio (V/C) at both the merge and non-merge sections. The variation in variations in lateral movement durations as a function of (V/C) ratio at merge and non-merge sections are shown in Figure 13 (a), (b), and (c).

- Lane changing rates (right and left lane changing) of vehicles due to merging of two different traffic streams at merge section are higher compared to a straight road section. To prove the significance a statistical t-test was conducted, and the results are shown in Table 8. From the t-test results it is clear that there exists a significant variation in lane change duration of cars (p value = 0.0117 < 0.05), heavy vehicles (p value = 0.0276 < 0.05), and motorcycles (p value = 0.0412 < 0.05) at merge section compared to non-merge section.
- Also, the reduction in speeds of vehicles (as shown in Table 3) due to merging of two different traffic streams at merge section compared to a straight road section leading to increase in lateral movement durations of different vehicle types.

# Lateral clearance

Further analysis in the lateral movement behavior of different vehicle-type combinations is carried out. The interactions between different vehicle types in the traffic stream, their lateral positions across the road section, and their lateral movement patterns are indicated by their lateral clearances. Lateral clearance depends on the vehicle type, its speed, and the speed of the adjacent vehicle. Computation of overtaking maneuver depends upon the lateral clearance parameter. In this study, the limits for lateral clearances for different vehicle types having different velocities are adopted



Figure 12. Total observations on lateral movement durations of different vehicle types.

from the work of Bangarraju et al. (Bangarraju, Ravishankar, and Mathew T 2016). From their work the minimum and maximum limits of lateral clearance values from speeds ranging from zero to 80 km/hr are implemented for computation of lateral clearance for different vehicle types. Figure 14 shows the schematic

 Table 7. Mean lateral movement duration for different leader-follower pairs.

			Lateral movement duration (s)						
Follower-	Dataset			Standard					
leader pair	size	Mean	Median	deviation	Minimum	Maximum			
C-C	648	5.2	5.0	1.9	1.5	14.5			
C-H	221	5.8	6.2	2.8	1.5	14.5			
C-M	426	4.6	4.2	3.1	1.5	14.5			
H-C	196	8.2	8.7	2.4	2.5	16.5			
H-H	114	9.4	8.9	3.3	2.5	16.5			
H-M	187	7.3	7.0	1.8	2.5	16.5			
M-C	385	3.0	2.5	1.2	0.6	10.5			
M-H	154	2.1	1.4	1.0	0.6	10.5			
M-M	337	1.5	1.2	2.2	0.6	10.5			

C-Car, H- Heavy vehicle, M- Motorcycle

representation of lateral clearance and lateral gaps between different vehicle types.

The lateral gap denotes the sideway spacing between vehicle bodies in a traffic stream whereas Lateral clearance is the actual physical space available between different vehicle types (Llorca et al. 2017).

The lateral clearance values for different vehicle types are formulated using equation (9) (Gowri and Sivanandan 2015).

$$C = \frac{C_{max} - C_{min}}{V_{max} - V_{min}} (V_c - V_{min}) + C_{min}$$
(9)

Here C denotes lateral clearance,  $C_{max}$  denotes maximum lateral clearance,  $C_{min}$  denotes minimum lateral clearance,  $V_{max}$  denotes maximum velocity,  $V_{min}$  denotes minimum velocity,  $V_c$  denotes current velocity.

The variation in lateral clearance is assumed to be linearly dependant on the vehicles speed. Table 9 shows the values of maximum and minimum lateral clearance maintained between different vehicle types.

 $C_{max}$  – maximum lateral clearance in m,  $C_{min}$  – minimum lateral clearance in m.



Figure 13. Lateral duration vs V/C ratio for different vehicle types (a) cars, (b) heavy vehicles, (c) motorcycles.

Table 8.	t-test results for	variation in lat	eral movement	duration v	v.r.t lane-cha	anging te	ndency	at merg	e and nor	n-merge	section.

		Merge section		N	on-merge sectio			
Vehicle type	Total observations (61% right and 39% left lateral movements)	Mean right lateral movement duration (s)	Mean left lateral movement duration (s)	Total observations (54% right and 46% left lateral movements)	Mean Right lateral movement duration (s)	Mean Left Lateral Movement Duration (s)	Aggregate Increase in Lateral Movement Duration (%)	p-value
Car	542	5.97	5.24	527	5.08	4.42	18	0.0117
Heavy vehicle	211	9.24	8.52	208	8.18	7.14	15.9	0.0276
Motorcycle	397	3.28	2.82	442	2.39	2.06	14.6	0.0412



Figure 14. Schematic representation of lateral clearance and lateral gaps between different vehicles.

 $\ensuremath{\text{Table 9.}}$  Minimum and maximum lateral clearances maintained by different vehicle types.

	Car		Tr	uck	Motorcycle		
Vehicle type	C <sub>min</sub>	C <sub>max</sub>	C <sub>min</sub>	C <sub>max</sub>	C <sub>min</sub>	C <sub>max</sub>	
Car	0.6	1.0	0.6	1.1	0.4	0.8	
Truck	0.6	1.1	0.6	1.2	0.4	0.9	
Motorcycle	0.4	0.8	0.4	0.9	0.2	0.6	

From the computed velocities of different vehicle types and using their lateral clearance limits, the variation in lateral clearance lateral clearance maintained by different leader-follower pair combinations (car-car, heavy vehicle-car, motorcycle-car, etc.) is evaluated as a function of their speeds, as shown in Figure 15 (a), (b), and (c). The lateral clearance with surrounding vehicles is comparatively higher if one of the leader-follower pair consisted of a heavy vehicle due to their broad static characteristics. However, in the case of motorcycle pairs, the lateral clearance is relatively lower due to their small size and high maneuvering capabilities.

#### Lateral gaps

The trajectory data and vehicle dimensions are utilized to extract the lateral gap data. The influence of macroscopic characteristics (volume/capacity ratio and percentage of Area-Occupancy) of the vehicles on the lateral gap maintenance of different leader-follower pairs are evaluated. A total of 755 datasets of different leader follower pairs are analyzed at both merge and non-merge sections and the variations in lateral gap maintenance behavior of different vehicle pairs as a function of volume to capacity ratio at merge and non-merge sections are presented in Figures 16 and 17 respectively.

- The lateral gap maintenance between different vehicle pairs decreased with increase in traffic inflow.
- The lateral gap is relatively higher in case if one of the vehicles in leader-follower pair is a heavy vehicle.
- Motorcycles maintained relatively lower lateral gaps compared to other vehicle pairs due to their lower static and higher maneuverable capabilities.

• The lateral gap maintenance between different vehicle-pairs is higher at non-merge section compared to merge section due to significant increase in speed between vehicle pairs.

Area-Occupancy is defined as the proportion of time the observed set of vehicles occupy a stretch of road section under study (Mallikarjuna and Rao 2006). Area-Occupancy is formulated as follows:

$$Area - Occupancy = \frac{\sum_{i} t_{i}a_{i}}{TA}$$
(10)

Here  $a_i$  denotes the area occupied by vehicle*i* at time  $t_i$  in m<sup>2</sup>,  $t_i$  denotes the time during which a vehicle *i* occupies the study section in s, *T* is the overall observation time in s, and *A* is the area of the entire road stretch in m<sup>2</sup>.

The variations in lateral gap maintaining behavior of different vehicle pairs are evaluated as a function of percentage vehicle occupancy at both merge and non-merge sections. The increase in occupancy of the vehicles has an inverse effect on the average lateral gap maintaining behavior for different vehicle types, as shown in Figure 18 (a),(b),(c) and Figure 19 (a),(b),(c).

- The average lateral gap of vehicle pairs consisting of heavy vehicles is relatively higher and motorcycles maintained relatively lower compared to other vehicle pairs at both merge and non-merge sections for an overall observed area occupancy of 80%.
- The average lateral gap maintenance between different vehicle types at non-merge section is comparatively higher compared to merge section since the speeds of different vehicle types at merge section are comparatively lower than their speeds at non-merge section.

#### **Overtaking behavior of vehicles**

The overtaking behavior of different vehicle types was analyzed from relevant field data extracted from video recording. An overtaking section of considerable length was chosen at merge section. The overtaking maneuver of vehicles is illustrated in Figure 20.



(c)

Figure 15. Lateral clearance versus speed for different leader-follower pairs at merge section (a) follower-car (b) follower- heavy vehicle (c) follower-motorcycle.



Figure 16. Lateral gap vs V/C ratio speed for different leader-follower pairs at merge section (a) follower-car (b) follower- heavy vehicle (c) follower-motorcycle.

Assuming subject vehicle X moving at a speed  $v_x$  (km/hr) at the time of initiation of overtaking maneuver. Y is the slow-moving vehicle in front of overtaking vehicle X, moving with uniform speed of  $v_y$  (km/hr).

Taking into assumption that when subject vehicle X is at position  $X_1$ , it reduces its speed to the speed of the slow-moving leader vehicle Y. The subject vehicle X from position  $X_2$  will start accelerating and moves to adjacent lane, overtaking the vehicle Y, and then maneuvers back to its initial lane ahead of Y at position  $X_3$  in time T. In the whole overtaking operation, the distance covered by subject vehicle X from  $X_2$  to  $X_3$  is represented by  $d_2$  in time T. The minimum longitudinal headway between the positions  $X_2$  and  $Y_1$  is taken as minimum spacing s between the vehicles X and Y both moving with the speed  $v_y$ . The minimum spacing between the vehicles during the initiation of overtaking maneuver is formulated as

$$s = 0.7v_y + 6$$
 (11)

This minimum spacing *s* is also assumed to be the minimum longitudinal headway between the positions  $Y_2$  and  $X_3$ . Time *T* denotes the overtaking maneuver of subject vehicle X from position  $X_2$  to  $X_3$ . During this time *T* the distance *y* covered by slow moving vehicle Y moving with a speed  $v_y$  is formulated as

$$y = v_y T \tag{12}$$

Therefore, the distance  $d_2$  is formulated as

$$d_2 = y + 2s \tag{13}$$

The speed of overtaken vehicle Y and the acceleration of overtaking vehicle X effects the overtaking operation time *T*. The formulation of overtaking maneuver time *T* is also done by equating the distance  $d_2$  to the general equation used to formulate the distance covered by a vehicle moving with uniformly acceleration with initial speed  $v_y$ , and acceleration *a*.

$$d_2 = y + 2s = v_y T + \frac{aT^2}{2} \tag{14}$$

$$y = v_y T, 2s = \frac{aT^2}{2} \tag{15}$$

$$T = \sqrt{\frac{4s}{a}}, s = 0.7v_y + 6$$
 (16)

$$d_2 = v_y T + 2s \tag{17}$$

The time taken for complete overtaking maneuver is evaluated from the data extracted using video analysis. The acceleration of overtaking vehicle is formulated from equation (18) as follows:

$$a = \frac{4s}{T} \tag{18}$$

Also, the speed of the overtaking vehicle X is formulated from the evaluated distance  $d_2$  and overtaking maneuver time T as follow:



Figure 17. Lateral gap vs V/C ratio speed for different leader-follower pairs at non-merge section (a) follower-car (b) follower-heavy vehicle (c) follower-motorcycle.

$$v_x = \frac{d_2}{T} \tag{19}$$

#### **Overtaking distance**

In total 303 and 256 overtaking maneuvers were observed between different vehicle types at both merge section and non-merge section. The total distance traveled by each vehicle type is presented in Table 10. At merge section the mean overtaking distance for heavy vehicles (73.5 m) is comparatively higher than other vehicle types due to their larger size and lower operating capabilities. It is also observed that the mean overtaking distance for motorcycles (34.2 m). at non-merge section the mean overtaking distance for heavy vehicles (92.3 m) is comparatively higher than other vehicle types due to their larger size and lower operating capabilities. It is also observed that the mean overtaking distance for motorcycles (34.2 m).

The variations in overtaking distance of different vehicle types at both merge and non-merge section are analyzed as a function of volume to capacity ratio. Figure 21 (a), (b), and (c) shows the variations in overtaking distance of different vehicle types as a function of different levels of traffic congestion.

- The overtaking distance of cars and heavy vehicles increased with increase in traffic inflow due to their reduction in speed till there is no scope for any overtaking due to increase in traffic density.
- Motorcycles overtaking distance also increased with increase in traffic inflow, however the overtakings took place even under congested conditions where V/C > 0.8 showing filtering

behavior through the congested traffic stream. This is due to the fact that motorcycles have high maneuverable capabilities owing to their smaller size.

- The overtaking distance of motorcycles reduced above the congested state (V/C > 0.8) since other vehicle types were moving with lower speeds or almost came to halt positions.
- Overall, there is an increase in overtaking distance of all vehicle types at non-merge section compared to merge section due to increase in speed of traffic stream and reduction in lane changes due to merging of two different traffic streams.

# **Overtaking time**

The total time required to pass a vehicle depends upon the relative speed, the space headway, and the presence of surrounding vehicle types. Table 11 presents the total overtaking time for each vehicle type at both merge and non-merge sections. At merge section the mean overtaking time for heavy vehicles (8.6 s) is comparatively higher than other vehicle types due to their larger size and lower operating capabilities. It is also observed that the mean overtaking time for heavy vehicles (10.2 s) is comparatively higher than other vehicles (10.2 s) is comparatively higher than other vehicles (10.2 s) is comparatively higher than other vehicle types due to their larger size and lower operating capabilities. It is also observed that the mean overtaking time for heavy vehicles (10.2 s) is comparatively higher than other vehicle types due to their larger size and lower operating capabilities. It is also observed that the mean overtaking time for motorcycles (5.2 s).

The variations in overtaking time of different vehicle types at both merge and non-merge sections are analyzed as a function of volume to capacity ratio. Figure 22 (a), (b), and (c) shows the



Figure 18. Average lateral gap vs area occupancy at merge section.

variations in overtaking time of different vehicle types as a function of different levels of traffic congestion.

- The overtaking time of cars and heavy vehicles is also directly proportional to traffic inflow due reduction in speed till there is no scope for any overtaking due to increase in traffic density.
- Motorcycles overtaking time also increased with increase in traffic inflow, however the overtakings took place even under congested conditions where V/C > 0.8 showing filtering behavior through the congested traffic stream. This is due to the fact that motorcycles have high maneuverable capabilities owing to their smaller size.
- The overtaking time of motorcycles reduced above congested state (V/C > 0.8) since other vehicle types were moving with lower speeds or almost came to halt positions.
- Overall, there is an increase in overtaking time of all vehicle types at non-merge section compared to merge section due to increase in speed of traffic stream and reduction in lane changes due to merging of two different traffic streams.

#### Variation of overtaking distance to overtaking time

From the analyzed datasets, the overtaking time increases with the increase in the overtaking distance. Low overtaking time means the vehicle is moving at higher velocity, hence lower overtaking distance. A graph was plotted from the observed data at the merging location for all the vehicle types, drawing a relationship between overtaking time and overtaking distance, as shown in Figure 23 (a), (b), and (c). From the graphs, it can be suggested that a linear

relationship exists between the two variables showing they are strongly correlated for all the vehicle types.

The variation in overtaking distance with speed difference between the vehicles involved in overtaking maneuvers are analyzed with varying levels of traffic inflow as shown in Figure 24 (a), (b), (c).

# Variation of overtaking time to speed difference

The dynamic characteristics of an overtaken vehicle such as its speed are the determining factor for overtaking distance and overtaking time. Hence the determination of the speed difference or the relative velocity between the overtaking and overtaken vehicles is necessary. The difference in velocities of all vehicle types to the overtaking time is plotted, as shown in Figure 25 (a), (b), and (c). The overtaking time increases with reduction in speed difference since when two vehicles move with equivalent speed, it causes difficulty for the overtaking vehicle to immediately overtake as the driver must maintain a minimum gap to prevent a collision. However, if the speed difference is significant than the slowmoving vehicle can quickly be passed by the fast-moving vehicle within a short time.

The variation in overtaking time with speed difference between the vehicles involved in overtaking maneuvers are analyzed with varying levels of traffic inflow as shown in Figure 26 (a), (b), (c).

# Effect of traffic flow on number of overtaking

Traffic volume has a profound impact on the number of overtaking maneuvers, especially at the merging location since two different traffic streams having different mean speeds converge at a point.

![](_page_20_Figure_1.jpeg)

Figure 19. Average lateral gap vs area occupancy at non-merge section.

![](_page_20_Figure_3.jpeg)

Figure 20. Illustration of overtaking maneuver of a vehicle.

Table 10. Overtaking distance of different vehicle types at both merge and non-merge section.

	Total overtaking distance (m)									
	Merge section				Non-merge Section					
Vehicle type	Mean	Minimum	Maximum	Std. Dev.	Mean	Minimum	Maximum	Std. Dev.		
Car	57.4	33.3	123.5	26.5	78.6	40.2	159.4	28.2		
Heavy Vehicle	73.5	41.8	148.5	39.2	92.3	55.6	186.6	43.3		
Motorcycle	34.2	24.4	101.6	24.8	49.4	31.4	112.8	22.8		

Table 11. Overtaking time for different vehicle types at merge section.

Vehicle type	Total overtaking time (s)									
		Merg	e section		Non-merge Section					
	Mean	Minimum	Maximum	Std. Dev.	Mean	Minimum	Maximum	Std. Dev.		
Car	4.5	3.6	15.5	2.1	6.8	3.8	17.7	1.8		
Heavy vehicle	8.6	5.8	20.4	3.2	10.2	6.4	22.5	2.7		
Motorcycle	4.0	3.2	12.6	2.0	5.2	3.4	14.8	2.3		

![](_page_21_Figure_0.jpeg)

Figure 21. Overtaking distance vs V/C ratio at merge and non-merge sections a) cars, b) heavy vehicles, and c) motorcycles.

The tendency to pass the slow-moving vehicle by fast-moving vehicle increases with an increase in the traffic flow up to the capacity of the road section. Further increase in traffic flow above the traffic capacity of the road reduces the number of overtaking due to the formation of a bottleneck, causing a reduction in the safe overtaking gaps between the vehicles. These conclusions can be observed in Figure 27.

# Summary and conclusion

This research analyses the traffic flow parameters at the merging location of a multi-lane urban road in Kuala Lumpur, Malaysia. This merging section is selected as it acts as a representative for all the remaining merging sections spread across the city. Very few studies were related to the parametric evaluation of traffic flow at merge sections on high-speed urban roads. Therefore, this study is carried out to evaluate the traffic flow parameters at the merging location on a five-lane divided urban highway and compared with a non-merge section. The traffic macroscopic and microscopic parameters such as speed, flow, deceleration rate, headway, and overtaking are evaluated at the merge and non-merge locations. The key results of this research are:

- Free speed distribution study carried out at the merging location showed a reduction in the speed of different vehicle types by 7–13% in the merge section compared to a non-merge. Heavy vehicle speeds are greatly affected by the increase in traffic flow. However, two-wheelers showed comparatively a slow pace in speed reduction, suggesting a vast number of within lane movements and filtering through the traffic phases.
- The following mean headway for motorcycles is comparatively lesser than other vehicle types, showing high manoeuvrability due to their small size. Hence these results show the presence of the car-following behavior that vary for different vehicle types.
- In both merge and non-merge sections, motorcycle combination pairs maintained the least space headway due to their smaller dimensions and higher maneuverable capabilities, followed by car and truck combination pairs.
- Huge variations can be found in the reaction time for different leader-follower pairs. Motorcycles have an average reaction time of 0.90 s, which is the least among the vehicle types. Moreover, it is also observed that slow-moving vehicles have a higher reaction time.
- There exists a significant difference in deceleration rates of different vehicle types based on their static and dynamic capabilities. High deceleration rates are observed between heavy vehicle pairs due to their large static and lower maneuvering capabilities.
- All vehicle types have varied lateral maneuvering capabilities because of a wide range of variations in their static and dynamic capabilities. These durations vary as high as 9 s (heavy vehicles) to as low as 5 s (motorcycles).
- The direction of lateral movement influences the duration of lateral shift. From the estimated results, the mean right lateral movement duration is 2.5*s* which is higher than the left lateral duration of 2.1*s*. This is due to the fact that the traffic speed is higher at the right lanes in Malaysia.
- The average lateral gap between the vehicles reduces with increase inflow. Also, the increase in occupancy of the vehicles has an inverse effect on the average lateral gap maintaining behavior.
- The overtaking time is directly proportional to the overtaking distance. Lower overtaking time means the vehicle is moving at higher velocity, hence lesser overtaking distance. A linear relationship exists between the two variables showing they are strongly correlated for all the vehicle types.
- The overtaking time increases with reduction in speed difference. When two vehicles move at same speed, it is challenging for the overtaking vehicle to immediately overtake as the driver must maintain a minimum gap to avoid a collision.

![](_page_22_Figure_1.jpeg)

Figure 22. Overtaking time vs V/C ratio at merge and non-merge sections a) cars, b) heavy vehicles, and c) motorcycles.

![](_page_22_Figure_3.jpeg)

Figure 23. Overtaking time vs overtaking distance (a) cars (b) heavy vehicles (c) motorcycles.

![](_page_23_Figure_1.jpeg)

Figure 24. Overtaking distance vs speed difference for varying V/C ratio (a) V/C  $\leq$  0.30, (b) 0.30 < V/C  $\leq$  0.60, (c) 0.60 < V/C  $\leq$  0.90.

![](_page_23_Figure_3.jpeg)

Figure 25. Overtaking time vs speed difference (a) cars (b) heavy vehicles (c) motorcycles.

![](_page_24_Figure_1.jpeg)

Figure 26. Overtaking time vs speed difference for varying V/C ratio (a) V/C  $\leq$  0.30, (b) 0.30 < V/C  $\leq$  0.60, (c) 0.60 < V/C  $\leq$  0.90.

![](_page_24_Figure_3.jpeg)

Figure 27. Number of overtakings vs traffic inflow.

- Motorcycles overtaking time increased with increase in traffic inflow; however the overtaking took place even under congested conditions where V/C > 0.8 showing filtering behavior through the congested traffic stream. This is due to the fact that motorcycles have high maneuverable capabilities owing to their smaller size.
- The tendency to out distance the slow-moving vehicle by fastmoving vehicle increases with an increase in the traffic flow up to the capacity of the road section. Further increase in traffic flow above the traffic capacity of the road reduces the number of overtaking due to the formation of a bottleneck, causing a reduction in the safe overtaking gaps between the vehicles.

Merging sections have a potential effect on the traffic flow movements on high-speed urban roads since they are the sources for bottleneck locations which might lead to traffic crashes due to the merging of different traffic streams. Their operations affect the efficiency and safety of urban road networks. The results from this present research help in the operational analysis of merging location on five-lane divided roads with familiar merging maneuver prevailing on various other merging sections on five-lane urban roads in Malaysia.

# **Contribution of the study**

This study revolves around the drivers' behavioral modeling under heterogeneous traffic flow features existing in Southeast Asian countries. By incorporating vehicle-type dependent factor, models were built and executed in a semi-discrete framework to model the behavioral interactions of different vehicles at merge and nonmerge sections under mixed traffic conditions, offering better vehicle movement forecasting and evaluation accuracy. By evaluating the trajectory data of vehicles traversing in heterogeneous traffic stream, the impact of vehicle type and non-lane-based movements on driver behavior at merge and non-merge sections is established. The outcomes of this study will be helpful in understanding the lateral, longitudinal, and overtaking behavior of different vehicle pairs at an urban high-speed merge section under heterogeneous traffic stream. This study also helps in the development of a simulation model for merging part on a multi-lane urban highway. The results support the level of service and safety aspects at merging locations with the event of a simulation model for various traffic flow situations. The changes in road geometrics and its aftereffects included in the simulation model provides a better understanding of the merging locations.

# Limitations of the study

The current study is limited to a five-lane merge and non-merge (mid-block) sections in plain terrain. The data collection in this study took place on a bright sunny weekday during summer season. This study of behavioral movements of different vehicle types can be extended to different road sections having different geometric conditions (horizontal and vertical curved roads, signalized and unsignalized intersections etc.). Also, parameters such as socio-economic factors, weekend traffic variations, variations in night-time traffic flow, monthly and seasonal variations in traffic flow affecting behavioral movements of vehicles will be considered.

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