Optimized Numerical Investigation of Heat And Mass Transfer in Porous Media

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Abstract:

The conjugate heat transfer phenomenon in porous medium, has gained considerable attention by many researchers* [1-8] *recently, due to the vast range of its applications in several areas of Engineering such as, Bio Instrumentation, Mechatronics and Bio Medical Engineering etc. This phenomenon is governed by set of nonlinear partial differential equations which are coupled in nature and can't be solved directly; hence a numerical method called finite element method is employed to solve them, which in turn generates a group of algebraic equations to put together in the form of a matrix and are solved by using an algorithm with a tight convergence criteria. The current research is aimed to discuss an optimized algorithm based on numerical investigation to solve these equations, and compare with the conventional algorithm. Research results shows significant improvements with proposed algorithm, which outperforms efficiently for any complex solid geometry compared to conventional algorithm, while conventional can only perform better for simple solid geometry.

Key words

Heat Transfer, Finite Element Method, Porous Media, Optimization, Partial Differential Equations

I. INTRODUCTION

The development of fast and efficient optimization algorithms[9, 10] have become inevitable in almost every single department of life, which has led to the incredible emphasis on increased awareness and creative ideas in developing the optimization techniques. This trend has profound effects, in particular, on research and innovation as well as solution techniques adopted for solving various scientific and industrial problems in the area of computer science as evident from the open literature. Furthermore, in engineering the optimization[11] techniques have become customary and often indispensable especially in actively pursued research area. However with the rapid advancement in computer science and information technology, it has become comparatively, easily achievable task. Therefore it comes as no surprise, as the emergence of interdisciplinary research collaborations are considered as the most effective tools in research and innovation fields, such as; bio instrumentation engineering, mechatronics engineering, bio medical engineering and so on. In a similar trend there is a need to develop optimized yet efficient and faster algorithms to investigate the various issues in heat transfer study. To accomplish this need of efficient and optimized[12] algorithmic tools in view, which could be more helpful in obtaining the solutions to the non-linear partial differential equations with acceptable accuracy in minimum time. In particular the conjugate heat transfer phenomenon, which has more complicated equations to handle, require the best possible solution technique that can execute with high efficiency and minimal computation time to achieve a desirable accuracy.

II. BACKGROUND

The emergence of computational technological advancement has played a very crucial role in understanding and solving many of the unsolved mathematical problems, helping us to understand the complex physical phenomenon which otherwise could have been left unanswered. Conjugate heat transfer in porous medium is one such phenomenon whose understanding was improved after the arrival of computer and advancement of computational methods.

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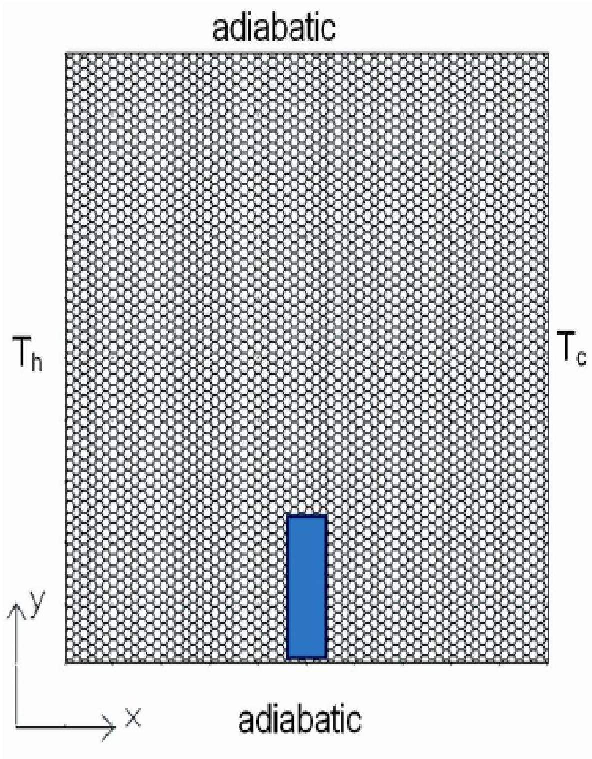

Fig. 1 ─ Porous cavity with solid [13]

The investigation of heat transfer through porous medium has received incredible attention from the various eminent researchers [14-19] due to its, prevalence in the various engineering and industrial applications and its complexity. The difficulty in dealing with the governing equations to study the above phenomenon has given rise to the implementation of the various numerical techniques to solve these nonlinear partial differential equations. The need for the solutions to the governing nonlinear partial differential equations has motivated to develop various verities of optimized numerical techniques. The researchers have suggested various optimization techniques to deal with these complex analyses as in the case of the study carried out by [9] and [10].

The present paper deals with the solution of the governing non linear partial differential equations to investigate the heat transfer characteristics and fluid flow pattern in a square porous domain fixed with solid at arbitrary position. The study is accomplished by using the most popular numerical technique Finite element Method FEM in which the partial differenitail equations are converted in to the simple algebariac equations by means of Galerkin's method. Furthermore these algebraic equations are solved for various physical and geometrical parameters by using the Matlab codes, generated to understand the effects of various physical and geometrical parametrs on the heat transfer and fluid flow behaviour. Many computer codes, for instance, physical phenomnon as mentioned in above study, consumes too much time and needs advanced technology to handle the robust analysis [11]. Thus it needs an efficient, in fact a specialized, optimized algorithm to circumvent the unnecessary huge time and resources to carry out the analyses.

III. METHODOLOGY:

The present paper discusses the proposed algorithm, solved using popular numerical method called finite element method [20-22]. The aforesaid phenomenon is governed by 4 nonlinear partial differential equations [23] viz., equation for a momentum, equation for energy in porous medium, equation for energy in solid and equation for concentration, respectively. This in turn forms mathematical representation of 4 equations model for conjugate heat and mass transfer phenomenon. Since the equations are coupled and are needed to be solved simultaneously; hence it makes them interdependent making the whole set of equations very complex and tricky. The set of 4 governing equations of problem under study are as stated below:

Equation for Momentum

$$\frac{\partial^2 \overline{\psi}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{\psi}}{\partial \overline{y}^2} = -Ra\left[\frac{\partial \overline{T}}{\partial \overline{x}} + N\frac{\partial \overline{C}}{\partial \overline{x}}\right] \tag{1}$$

Energy equation for porous medium

$$\frac{\partial \overline{\psi}}{\partial \overline{y}}\frac{\partial \overline{T}}{\partial \overline{x}} - \frac{\partial \overline{\psi}}{\partial \overline{x}}\frac{\partial \overline{T}}{\partial \overline{y}} = \left(\left(1+\frac{4R_d}{3}\right)\frac{\partial^2 \overline{T}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{T}}{\partial \overline{y}^2}\right) \tag{2}$$

Solid region Energy equation

$$\left(1+\frac{4R_d}{3}\right)\frac{\partial^2 \overline{T_s}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{T_s}}{\partial \overline{y}^2} = 0 \tag{3}$$

Equation for Concentration

$$\frac{\partial \overline{\psi}}{\partial \overline{y}}\frac{\partial \overline{C}}{\partial \overline{x}} - \frac{\partial \overline{\psi}}{\partial \overline{x}}\frac{\partial \overline{C}}{\partial \overline{y}} = \frac{1}{Le}\left(\frac{\partial^2 \overline{C}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{C}}{\partial \overline{y}^2}\right) \tag{4}$$

Equations 1-4 are non-dimensionalised using the basic form of equations as presented in [24-27] . As stated earlier, these equations are difficult to solve directly, hence by applying Finite Element Method, equations 1 – 4 are changed into matrix form of equations as stated below:

$$\frac{1}{4A}\left\{\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_2 & b_2b_3 & b_3^2 \end{bmatrix}+\begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_2 & c_2c_3 & c_3^2 \end{bmatrix}\right\}\begin{Bmatrix} \overline{\psi}_1 \\ \overline{\psi}2 \\ \overline{\psi}3 \end{Bmatrix}=\frac{Ra}{6}\left[\begin{Bmatrix} b_1\overline{T}_1+b_2\overline{T}_2+b_3\overline{T}_3 \\ b_1\overline{T}_1+b_2\overline{T}_2+b_3\overline{T}_3 \\ b_1\overline{T}_1+b_2\overline{T}_2+b_3\overline{T}_3 \end{Bmatrix}+N\begin{Bmatrix} b_1\overline{C}_1+b_2\overline{C}_2+b_3\overline{C}_3 \\ b_1\overline{C}_1+b_2\overline{C}_2+b_3\overline{C}_3 \\ b_1\overline{C}_1+b_2\overline{C}_2+b_3\overline{C}_3 \end{Bmatrix}\right]$$

(5)

$$\frac{1}{12A}\begin{Bmatrix} c_1\overline{\psi}_1+c_2\overline{\psi}_2+c_3\overline{\psi}_3 \\ c_1\overline{\psi}_1+c_2\overline{\psi}_2+c_3\overline{\psi}_3 \\ c_1\overline{\psi}_1+c_2\overline{\psi}_2+c_3\overline{\psi}_3 \end{Bmatrix}\begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}-\frac{1}{12A}\begin{Bmatrix} b_1\overline{\psi}_1+b_2\overline{\psi}_2+b_3\overline{\psi}_3 \\ b_1\overline{\psi}_1+b_2\overline{\psi}_2+b_3\overline{\psi}_3 \\ b_1\overline{\psi}_1+b_2\overline{\psi}_2+b_3\overline{\psi}_3 \end{Bmatrix}\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}$$

$+$

$$\frac{1}{4A}\left[\left(1+\frac{4}{3}R_d\right)\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_2 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix}+\begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_2 & c_2c_3 & c_3^2 \end{bmatrix}\right]\begin{Bmatrix} \overline{T}_1 \\ \overline{T}_2 \\ \overline{T}_3 \end{Bmatrix}=0$$

(6)

$$\frac{1}{4A}\left[\left(1+\frac{4}{3}R_d\right)\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_2 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix}+\begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_2 & c_2c_3 & c_3^2 \end{bmatrix}\right]\begin{Bmatrix} \overline{T}_{1s} \\ \overline{T}_{2s} \\ \overline{T}_{3s} \end{Bmatrix}=0$$

(7)

$$\frac{1}{12A}\begin{Bmatrix} c_1\overline{\psi}_1+c_2\overline{\psi}_2+c_3\overline{\psi}_3 \\ c_1\overline{\psi}_1+c_2\overline{\psi}_2+c_3\overline{\psi}_3 \\ c_1\overline{\psi}_1+c_2\overline{\psi}_2+c_3\overline{\psi}_3 \end{Bmatrix}\begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}-\frac{1}{12A}\begin{Bmatrix} b_1\overline{\psi}_1+b_2\overline{\psi}_2+b_3\overline{\psi}_3 \\ b_1\overline{\psi}_1+b_2\overline{\psi}_2+b_3\overline{\psi}_3 \\ b_1\overline{\psi}_1+b_2\overline{\psi}_2+b_3\overline{\psi}_3 \end{Bmatrix}\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}$$

$$+\frac{1}{4A\,Le}\left[\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_2 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix}+\begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_2 & c_2c_3 & c_3^2 \end{bmatrix}\right]\begin{Bmatrix} \overline{C}_1 \\ \overline{C}\,2 \\ \overline{C}\,3 \end{Bmatrix}=0$$

(8)

Where, non-dimensional parameters are:

$$R_d=\frac{4\sigma T_c^3}{\beta_R k},\quad Ra=\frac{g\beta_T\Delta TKL_c}{\nu\alpha},\quad Le=\frac{\alpha}{D},\quad N=\left(\frac{\beta_c\Delta C}{\beta_T\Delta T}\right), \qquad (9)$$

Equations involved to solve phenomenon under study are coupled in nature meaning change in one equation affects the other and vice versa. Hence, they are solved in an iterative manner by setting suitable convergence criteria. The convergence criteria for all the variables are set as:

$$\left(\overline{\psi}^i_{nw}-\overline{\psi}^i_p\right)\le 10^{-7}$$
(10)

$$\left(\overline{T}^i_{nw}-\overline{T}^i_p\right)\le 10^{-5}$$
(11)

$$\left(\left(\overline{T}_s\right)^i_{nw}-\left(\overline{T}_s\right)^i_p\right)\le 10^{-5}$$
(12)

$$\left(\overline{C}^i_{nw}-\overline{C}^i_p\right)\le 10^{-5}$$

Where node i =*1….n* (13)

In the above expressions, *p* and *nw* represents previous and new, while in the domain, *n* and *i* represents total number of nodes and specific node number.

IV. Results and Discussion

The current problem domain initially is considered with a simple square mesh as show in figure 1, which contains 9 nodes and 8 elements, each node in the mesh is a point where we are supposed to find temperature and velocity profiles. In simple square mesh 9 nodes represents 9 points where we are supposed to solve 4 equations for conjugate heat and mass transfer phenomenon, which means a simple square mesh need 9 X 4 = 36 equations to solve. But unfortunately the physical problem geometries are not as simple as it is seen in case of simple mesh. So, as to solve the problem under study we need to employ a bigger mesh to get realistic results hence 36*36 matrix is considered. The mesh with 36 X 36 creates 2592 triangular elements having 1369 nodes, hence if we want to solve conjugate heat and mass transfer phenomenon using this conventional approach for the mesh (36 x 36) we need to solve 1369 x 4 = 5476 equations which are difficult to solve manually hence the algorithm is translated into a computer program using Matlab software. It is found that conventional algorithm ran into 1101 lines code, when translated into Matlab program. Numbers of equations increases considerably if we consider a bigger mesh.

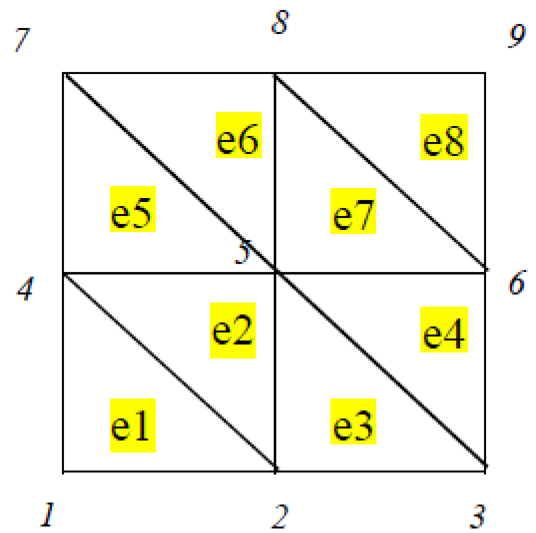

Figure 2: 8-element mesh[28]

By applying proposed algorithm we can reduce equations from 4 to 3 for the phenomenon under study and this is possible because equation 2 is similar to the right hand side of equation 3 and this can be done because $\overline{\psi}$ represents the velocity field in the porous medium and its value can be assigned to zero in the region occupied by solid wall, because there cannot be any fluid flow inside the solid, this reason avails us to reduce the number of equations from 4 to 3 which means the proposed algorithm when applied to a mesh (36 x 36) we need to solve 1369 x 3 = 4,107 instead of 1369 x 4 = 5476 equations which clearly reduces the number of equations by 1,369. Efficiency of proposed algorithm is validated using a case study whose results are listed below in table 1.

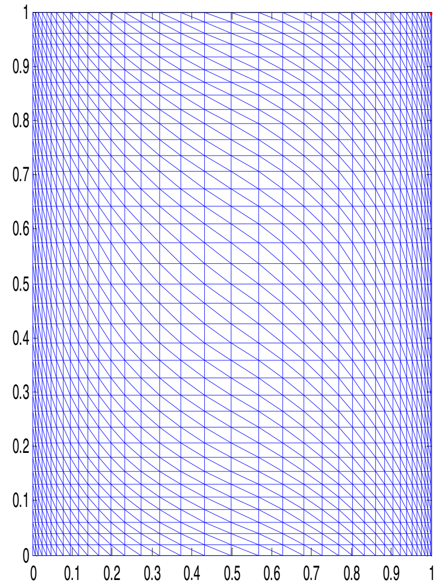

Figure 3: Mesh of physical domain[28]

Statistics of a conventional algorithm considering Figure 3, can be summarized in tabular format as show below:

Rows(R)	**Columns(C)**	**Elements**	**Nodes**	**Number of Equations**
36	36	(R*C)*2= 2592	(R+1)*(C+1)= 1,369	1,369

Phenomenon	**Number of Equations**
Conjugate heat and mass transfer	4 * 1369 = 5,476

Statistics of a proposed algorithm considering figure 3 under study, can be summarized in tabular format as show below:

Rows(R)	**Columns(C)**	**Elements**	**Nodes**	**Number of Equations**
36	36	(R*C)*2= 2592	(R+1)*(C+1)= 1,369	1,369

Phenomenon	**Number of Equations**
Conjugate heat and mass transfer	3 * 1369 = 4,107

Thus, from the above comparison it is evident that the proposed algorithm has been able to reduce 1,369 equations consider figure 2 mesh.

Elements of Porous Region	Elements of Solid Region	Analysis of proposed algorithm			Analysis of conventional algorithm		
		Number of Iterations	Time in seconds	Remarks	Number of Iterations	Time in seconds	Remarks
2520	72	396	2081.7	Converged	>500	2449	Unconverged
2448	144	268	1399.1	Converged	>500	2127.1	Unconverged
2376	216	203	1001.9	Converged	274	1194.9	Converged
2304	288	171	881.7	Converged	170	873.0	Converged
2592	0	48	169.40	Converged	48	172.16	Converged
0	2592	3	32.5286	Converged	3	33.3166	Converged

Table 1: Analysis of two Algorithms [28]

The approach of conventional and proposed algorithms results are compared and validated with existing literature [28]. The evaluation is done for two cases; with the absence of solid and with solid and porous medium together. The resultant analysis is shown in table 1, which is obviously justified for both the approaches.

V. CONCLUSION

Based on results achieved it is evident that the proposed algorithm out performed over the conventional approach as it solves conjugate heat and mass transfer with 3 equations whereas, conventional algorithm takes 4, similarly to solve conjugate heat transfer phenomenon proposed algorithm takes 2 equations conversely conventional algorithm takes 3. Proposed algorithm requires regular mesh such as square without any discontinuity. Whereas conventional algorithm requires two separate mesh's viz., for porous region and solid region. Proposed algorithm works well for any complex solid geometry conversely, conventional algorithm works well only for simple solid geometry only. Writing computer program code is relatively easy for proposed algorithm conversely it is quite involved for conventional algorithm, likewise proposed algorithm's program code length is relatively easier compared to conventional approach. Lastly it is found that computational resources required for proposed algorithm are relatively low compared to conventional method, which is evident from the data provided as of table 1.

Symbols used:

S. No	Symbol	Description
1	$C, \overline{C}$	concentration
2	$T, \overline{T}$	Temperature
3	g	Acceleration due to gravity (m/s^2)
4	K	Permeability of porous medium (m^2)
5	k_p , k_s	Porous and Solid thermal conductivity respectively (W/m-ºC)
6	$Kr = k_s/k_p$	Conductivity ratio
7	L	Height and length of cavity (m)
8	Le	Lewis number
9	Nu	Nusselt number
10	N	Buoyancy ratio
11	R_d	Radiation parameter
12	Ra	Modified Raleigh number
13	β_r	Absorption coefficient (1/m)
14	ψ	Stream function
15	$\overline{\psi}$	Non-dimensional stream function
16	α	Thermal diffusivity (m^2/s)
17	β	Coefficient of thermal expansion (1/ºC)
18	ρ	Density (kg/m^3)
19	ν	Coefficient of kinematic viscosity(m^2/s)
20	σ	Stephan Boltzmann constant (W/m^2-K^4)

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