A voice coil based electromagnetic system for calibration of a sub-micronewton torsional thrust stand

Jiang Kai Lam, Seong Chun Koay, Chie Haw Lim, Kean How Cheah
School of Engineering, Taylor’s University, Malaysia
Faculty of Engineering, University of Nottingham Malaysia Campus, Malaysia
School of Engineering and Physical Sciences, Heriot-Watt University Malaysia, Malaysia

Abstract
This paper presents the development of an alternative electromagnetic calibration system. Utilising commercially available voice coils and permanent magnets, the proposed system is able to generate linear, repeatable, and consistent steady-state calibration forces at over four orders of magnitude (30–23,000 μN). It is also capable of producing calibration impulse bits in the range of 12–668 μNs.

The maximum uncertainty errors of the calibrator are evaluated as 18.48% and 11.38% for steady-state and impulsive forces calibration, respectively. Its performance is compared to other existing electromagnetic calibration techniques. Capability of the system is then demonstrated in calibrating a sub-micronewton torsional thrust stand.

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1. Introduction
Nanosatellites (<10 kg) are finding new applications in various areas [1]. They are simpler and require shorter development time. Thus, they are inherently cost effective and ideal for demonstrating new and innovative ideas in outer space. In a nanosatellite, one of the desired sub-systems is micropropulsion system. The inclusion of microthrusters into nanosatellites is beneficial for improved operations in attitude control, station keeping, drag compensation, and orbital transfer [2]. Over the years, various microthrusters have been developed with the help of microelectromechanical systems (MEMS) [2–5].

Initially, there are three main configurations for pendulum thrust stand, i.e. hanging [7–9], inverted [10,11], and torsional [12–14], each with their own advantages and limitations.

Non-contact calibration systems include gas dynamic [17,18], electrostatic (ES) [15,19–21], and electromagnetic (EM) [13,22,23]. Gas dynamic calibrators are reliable in producing calibration forces between nanonewton and sub-micronewton [18]. In contrast, ES calibrators are able to provide a wider range of calibration forces, typically between hundreds of nanonewton and thousands of microneutron [21]. Nevertheless, this versatile calibration technique requires high voltage for generation of sufficiently large calibration force. This results in the need of more costly equipment, e.g. high voltage amplifier. For EM calibrators, the reported calibration forces are sub-micronewton and above. They exhibit good consistency and repeatability. Besides, EM calibrators are easier to be implemented compared to gas dynamic and ES. They mostly consist of an electromagnet (solenoid) coupled

* Corresponding author.
E-mail address: k.cheah@hw.ac.uk (K.H. Cheah).
with permanent magnet, current-carrying copper wire, or metal conductor. Unlike ES calibration system which is well established, EM calibration for torsional thrust stand is a relatively new idea. It can be further improved in terms of performance, as well as simplified for its implementation.

While the fundamental working principles remain straightforward and simple, effective yet commercially available components can be implemented in order to innovate EM as an alternative technique for thrust stand calibration. In this study, we explore the feasibility of using commercially available voice coils as an alternative EM calibration system for a sub-micronewton torsional thrust stand. Due to the nature of its construction, voice coil acts as a small and light weight electromagnet that can be utilised as part of the calibration system. Combining with a coin-sized permanent magnet, this newly developed calibration system is very compact. Compact calibration system is beneficial for integration with small testing facilities, in particular the vacuum chamber, in which the setup cost is proportional to the overall size.

The selected voice coil is first tested with different permanent magnets to investigate the characteristics of the EM calibration force generated. The calibration system is then implemented onto the thrust stand to showcase its performance as a calibrator for both steady-state and impulse forces. Lastly, the uncertainty error of the calibrator is evaluated and its performance is compared with other EM calibrators.

2. Torsional thrust stand setup

A thrust stand based on the working principle of torsional pendulum is designed and built in existing study. Generally, torsional pendulum thrust stand has good balance of high measuring and low vibrational noise sensitivities [6]. The dynamic motion of a torsional pendulum is described as:

\[
J\ddot{\theta} + \lambda\dot{\theta} + k\theta = F_t r_t
\]

where \( J \) is the moment of inertia about the rotational axis, \( \theta \) is the angular displacement of the pendulum, \( \lambda \) is the damping coefficient, \( k \) is the torsional elastic constant and \( F_t \) is the externally applied force at a distance of \( r_t \) from the rotational axis.

Our thrust stand consists of a 60 cm torsional arm made of U-shaped aluminium beam. It is light weight (210 g) yet sufficiently stiff to support external loadings mounted onto it. The torsional arm was supported by a single-ended flexural pivot (F-20, C-Flex) that acts as torsional spring. The pivot was clamped and connected to a heavy rectangular aluminium base (3 kg). Four anti-vibration mounts (126–3904, RS Pro) were installed to enhance its stability against external vibration. A strong permanent magnet was placed in close proximity under the torsional arm to induce an eddy current brake in order to dampen the oscillation of the arm. A high resolution (0.5 μm) laser displacement sensor (HL-G103-S-J, Panasonic) was positioned at one end of the torsional arm to measure its deflection. The EM calibrator was installed at the other end of the arm. The fluctuation of surrounding temperature and ambient air could affect the thrust stand response. The thrust stand was set up in an air-conditioned laboratory and enclosed with a transparent acrylic casing, where the temperature remains stable. The voice coil releases heat when excessively high current is applied. The electrical current was capped at 0.4 A and the voice coil was mounted externally. These precautionary steps minimise if not eliminate any significant heat transfer through the torsional arm that may cause undesired motion. The CAD drawing and actual setup of the thrust stand are shown in Fig. 1(a) and (b), respectively.

3. Electromagnetic calibration system

3.1. Voice coil and permanent magnet as electromagnetic calibrator

The EM calibration system used in this study consists of a voice coil and a permanent magnet, as shown in Fig. 2. Voice coil is commercially available in different sizes and commonly used in loud-speakers. It is essentially a solenoid, whereby an electromagnetic field can be generated when electrical current passes through the coil. Electromagnetic field strength of the voice coil, \( B \), is governed by Ampere’s Law:

\[
B = \mu_0 n l I
\]

where \( \mu_0 \) is the permeability of vacuum, \( n \) is the number of turns of wire per unit length, and \( I \) is the amount of electrical current flow through the coils.

In this study, electrical current of various levels were supplied to the voice coil in order to generate electromagnetic field of different strengths. The voice coil was then engaged to a permanent magnet to induce interactions between their magnetic fields. They were arranged in a way such that they repel each other. As a result,
it produces the electromagnetic force needed for the calibration of the thrust stand.

3.2. Electromagnetic force measurement using weighing balance

The amount of electromagnetic force generated by the EM calibration system was first measured experimentally. After a few trials, a 25.5 mm diameter voice coil was selected as it provides a wide range of calibration force. In addition, a voice coil of this size is compact thus easy to be set up. Two different types of permanent magnet, i.e. a ferrite disk magnet (weaker magnetic field strength, 100 Gauss) and a neodymium disk magnet (stronger magnetic field strength, 2000 Gauss) were used (see Fig. 3). By using magnets of different levels of magnetic field strength, the range of calibration forces becomes wider. As such, existing torsional thrust stand can be calibrated to characterize different types of micropropulsion systems.

The setup for electromagnetic force measurement is schematically shown in Fig. 4. A weighing balance (HR250AZ, A&D Weighing) with a resolution of 0.1 mg (0.981 μN) was used to measure the electromagnetic force. The permanent magnet was fixed onto the weighing balance while the voice coil was fixed externally to a mechanical stage (PT3/M, Thorlabs) for position and engagement adjustment. The permanent magnet was placed on top of a long plastic rod instead of directly on the weighing pan. This is to avoid any magnetic interaction that could affect the measurement. A power supply (MP303-3, Meguro) was used to supply electrical current to the voice coil. A current sensor (CTSR1-P, LEM) with a resolution of 10 mA and a digital oscilloscope (DS1102E, Rigol) were used to measure the amount of electrical current that flows through the coil. Electrical current ranges from 0.01 A to 0.4 A were supplied and the corresponding readings from the weighing balance were recorded.

The effect of engagement distance between the voice coil and the magnet on the stability of electromagnetic force has also been studied. This is to identify the acceptable range of engagement distance for generation of consistent and repeatable electromagnetic force. For this purpose, the current was fixed at three distinct levels (0.1 A, 0.25 A and 0.4 A) which represents low, mid and high range of forces, respectively. The engagement distance was varied from –3 mm to 3 mm using the mechanical stage. The actual setup is shown in Fig. 5.

3.3. Implementation onto torsional thrust stand

After the force measurement, the EM calibrator was installed onto the thrust stand as shown in Fig. 6. The permanent magnet was fixed to the other end of the torsional arm as opposed to the end with the laser displacement sensor. The voice coil was placed externally and its positioning and engagement to the magnet were adjusted using the mechanical stage. The electromagnetic forces were then applied to the torsional arm.

Steady-state force and impulsive force were applied for the calibration. For steady-state force calibration, the operational procedure is similar to the description in Section 3.2, where the same power supply was used to provide electrical current of various levels through the voice coil. As for impulsive force calibration, the power supply was replaced with a function generator (FG-8202, Dagatron) for generating impulse bits of various amplitudes and pulse widths. As the calibration forces were applied, the

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**Fig. 3.** Specifications of voice coil (left) and magnets (right) used.

**Fig. 4.** Schematics of EM force measurement setup and arrangement of voice coil and magnet.

**Fig. 5.** Actual experimental setup for EM force measurement. Inset picture shows a close-up view of the EM calibrator on the weighing balance.
corresponding deflections of the arm were recorded using the linear displacement sensor.

4. Results and discussion

4.1. Electromagnetic force measurement

Using the setup shown in Fig. 5, the force generated can be measured. Fig. 7 shows the electromagnetic forces generated by the EM calibration system at varying electrical current levels. Overall, the neodymium magnet-based system generates much higher force than the counterpart of ferrite magnet. This is predominantly due to the difference in their magnetic field strengths. Neodymium magnet is well known for its strong magnetism. Thus, given the same amount of electrical current through the voice coil, the magnet with stronger field strength repels harder, thus the force generated is higher, and vice versa.

It is noted that the relationship between the electromagnetic force generated and the electrical current supplied is linear and directly proportional. This is in agreement with previous finding [22] which proposed the electromagnetic force produced, \( F_{EM} \), is governed by:

\[
F_{EM} = B_m N \pi r_c I
\]

where \( B_m \) is magnetic flux density at the coil location, \( N \) is the number of coil turns, \( r_c \) is the radius of coil and \( I \) is the electrical current passes through the coil. The equation indicates that \( F_{EM} \) is directly proportional to the amount of \( I \) supplied. \( N \) and \( r_c \) are fixed as only the 25.5 mm voice coil was used. \( B_m \) is related to the engagement distance between voice coil and magnet as its magnitude is affected by the location of the two components, which in turns affect \( F_{EM} \).

Such relationship indicates that the electromagnetic force generated using the proposed EM calibrator is predictable and reliable as long as the \( B_m \) stays relatively constant. This is crucial for its applications onto the torsional thrust stand later on. The combination of the two magnets has an extended range of electromagnetic forces that covers four orders of magnitude, i.e. 30–2200 \( \mu \)N and 920–23,000 \( \mu \)N, for ferrite magnet based and neodymium magnet based calibrator, respectively.

With the same setup as above, the change in electromagnetic force with the engagement distance for both ferrite and neodymium based calibrators were determined as shown in Fig. 8. Note that 0 mm distance (initial position) represents that the coiled edge of voice coil and the upper surface of magnet are in the same plane (see Fig. 4). For example, 1 mm adjustment means that the magnet is 1 mm apart from the voice coil edge while –1 mm adjustment means that the magnet is 1 mm into the voice coil edge.

An apparent trend observed is that the electromagnetic force decreases regardless of the current level and the polarity of the engagement distance (magnet separated from or into the voice coil). This indicates that if the magnet is too far away (both positive and negative distances) from the coiled edge of voice coil, the \( B_m \) becomes less uniform, which causes a reduction in magnetic field interaction between the voice coil and the magnet. As a result, the force generated also decreases, as outlined in Eq. (3).

Further investigation reveals that for all current levels tested, the percentage difference in deviation of forces generated exceeds 10\% at engagement distances beyond ±2 mm, which can cause negative impacts on the repeatability and consistency of the system. This serves as a guideline for installing the EM calibrator onto the thrust stand, where the engagement distance has been kept within ±2 mm for reproducible and consistent calibration force generation.

4.2. Steady-state force calibration

The motion of torsional thrust stand can be characterized using linear displacement sensor. When an external constant force, \( F_c \), is applied to the thrust stand at \( r_t \), its motion is described as:

\[
F_c r_t = k \Delta \theta
\]

where \( k \Delta \theta \) is the steady state angular displacement of the thrust stand caused by the force.

Considering the very small magnitude of rotation, angular displacement of the thrust stand can be approximated using linear displacement as:

\[
\Delta x = r_{LDS} \theta
\]

where \( r_{LDS} \) is the distance between the linear displacement sensor (LDS) and the rotational axis.

After the EM calibrator was installed as shown in Fig. 6, different levels of steady-state (constant) electromagnetic forces were generated and applied to the torsional arm. Fig. 9 shows the typical torsional arm response to the force applied.
Based on Eq. (6), a calibration curve of linear displacement against the calibration force was plotted, as shown in Fig. 10. By using the combined ferrite and neodymium magnet-based EM calibrator, the thrust stand is calibrated to resolve steady-state forces for four orders of magnitude i.e. 30–16,000 μN. The range of calibration force is sufficient for most micropropulsion systems with exception for some high-thrust solid propellant micropropulsion systems which could produce thrust force as high as mN. The resultant calibration force applied is a mere 0.4 μN when 0.1 A of current is supplied to the EM calibrator. This indicates that the system is stable with negligible zero drift.

Combining Eqs. (4) and (5), the relationship between constant calibration force, $F_{\text{cal}}$, and linear displacement measured by LDS, $\Delta x_{\text{LDS}}$, can be expressed as:

$$F_{\text{cal}} = k \frac{r_{\text{LDS}}}{T_{\text{LDS}}} \Delta x_{\text{LDS}}$$  \hspace{0.5cm} (6)

4.3. Impulsive force calibration

Maintaining the same setup as in Fig. 6, calibration for impulsive force measurement was done by applying various impulse bits to the thrust stand. To produce impulse bits, the EM calibrator was connected to the function generator with squared input signals. By adjusting the signal amplitude (amount of electrical current) and the pulse time, different levels of calibration impulse bit can be produced.

In order to obtain a linear relationship between thrust stand response and calibration impulse bit applied, the pulse time should be much shorter than the natural period of the thrust stand, $1/f_{\text{nat}}$ [19]. Ideally, the pulse time should be within one-tenth of the natural period [24]. Hence, the natural period of existing thrust stand was determined by using the Fourier transform of the displacement readings acquired under free motion for 1 min. The $f_{\text{nat}}$ was evaluated as 1.8 Hz (peak of the curve) from power spectral density curve as shown in Fig. 11. This implies that the pulse time should not exceed 55 ms to ensure linearity between thrust stand response and calibration impulse bit.

When an impulsive force, $I_{\text{bit}}$, is applied to the torsional thrust stand, the maximum angular displacement, $\theta_{\text{max}}$, is given by:

$$\theta_{\text{max}} = \frac{r_{\text{LDS}} I_{\text{bit}}}{J_{\text{LDS}}}$$  \hspace{0.5cm} (7)

where $\omega_{\text{d}}$ is the natural frequency of the torsional thrust stand.

Re-arranging Eq. (7) with the consideration of small angle approximation, the impulsive force can be related to linear displacement as:

$$I_{\text{bit}} = \frac{J_{\text{LDS}} \omega_{\text{d}}}{r_{\text{LDS}}} \Delta x_{\text{max}}$$  \hspace{0.5cm} (8)

An example of the squared signal produced using the function generator for the EM calibrator is shown in Fig. 12, where the pulse width is adjusted to 48 ms and the voice coil output signal amplitude is 0.18 V. The current sensor used has a sensitivity of 1.2 V/A, so the corresponding current reading is 0.15 A. From Fig. 7, 0.15 A produces 833 μN and 12,200 μN of force for ferrite based and neodymium based calibrator, respectively. The resultant calibration impulse bit can be determined by multiplying the pulse width with the forces, which gives 40 μNs for ferrite based and 585 μNs for neodymium based calibrator.

By adjusting the function generator, the range of calibration impulse bits produced is 12–668 μNs. Subsequently, they were applied to the thrust stand and the corresponding response (maximum deflection range, $\Delta x_{\text{max}}$) was recorded using the laser displacement sensor. Fig. 13 shows the typical thrust stand response to a calibration impulse bit of 585 μNs. The same impulse bit was repeated three times within a duration of 25 s. It can be observed that the maximum deflection range obtained (663 μm in this case) is very consistent with a standard deviation of 1.57 μm (0.24% of the measured value). This shows that the control...
of the pulses is precise and the signal itself is of good quality, resulting in consistent impulse bit generation.

Each maximum deflection range corresponding to the applied impulse bit is plotted to produce the calibration curve for impulsive force measurement, as shown in Fig. 14. The curve has shown good linearity. Furthermore, the calibration impulse bits from ferrite-based and neodymium-based EM calibrator overlap and the difference is within 5%, as shown in the inset graph. The good agreement in the overlapping region allows the thrust stand to be calibrated using ferrite-based EM calibrator at the lower range of impulse bits and switch to neodymium-based EM calibrator from medium range of impulse bits onwards until maximum allowable calibration impulse bits. After this calibration, the thrust stand can be used to measure impulsive forces from pulsed mode micropulsion systems, e.g. pulsed plasma thruster [25].

4.4. Uncertainty error analysis

Uncertainty in the EM calibrator system stems from the resolution of the instrumentations used. Aggregating the component errors contributed by each instrument, the uncertainty error in EM calibration can be calculated as:

\[
\frac{\Delta F}{F} = (1 + \beta)\sqrt{\left(\frac{\Delta I}{I}\right)^2 + \left(\frac{\Delta F_c}{F_c}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta t_c}{t_c}\right)^2 + \left(\frac{\Delta x}{x}\right)^2 + (\Delta F_{ch})^2}
\]

where 

- \(\Delta F\) is the uncertainty in the force measurement.
- \(F\) is the measured force.
- \(\Delta I\) is the uncertainty in the electrical current.
- \(I\) is the electrical current.
- \(\Delta F_c\) is the uncertainty in the calibration force.
- \(F_c\) is the calibration force.
- \(\Delta L\) is the uncertainty in the linear displacement.
- \(L\) is the linear displacement.
- \(\Delta t_c\) is the uncertainty in the pulse width.
- \(t_c\) is the pulse width.
- \(\Delta x\) is the uncertainty in the engagement distance.
- \(x\) is the engagement distance.
- \(\Delta F_{ch}\) is the uncertainty in the free motion displacement signal.

\(\beta\) is included as additional consideration for other unpredictable sources of error. It also accounts for the inaccuracy in estimating the known sources of error as summarized in Table 1.

Electrical current, \(I\), produced by the power supply used in this study has a resolution of 1 mA. The EM calibration forces, \(F_c\), were measured using a weighing balance of 0.1 mg resolution which is equivalent to 0.981 μN. The engagement distance, \(x\), between the permanent magnet and voice coil was adjusted using a mechanical stage with 10 μm resolution. The squared wave signal or pulse width, \(t_c\), was controlled on the function generator with an identical rise and fall time of 0.1 ms. The linear displacement sensor that
was used to measure the deflection, $\Delta x$, of the torsional arm has a resolution of 0.5 $\mu$m.

The characteristics of existing EM calibrator also contribute to the uncertainty error. They are the non-equivalence between two permanent magnets (up to 5%) and non-linearity of EM forces (up to 2%), as shown in Fig. 10. These factors are accounted for and consolidated into the variable of $D_F$.

Considering the known sources of error and a conservative value of 20% for $\beta$, uncertainty errors were computed. For steady-state calibration, the error is 18.47% for the minimum calibration force of 30 mN. A closer examination reveals that the power supply contributes the most for this considerably high error. In this case, 1 mA of resolution is rather low as the electrical power drew by the voice coil to generate the smallest calibration force is 10 mA. Nevertheless, the error has been reduced to 7.80% as the magnitude of calibration force increases. For impulsive calibration, the error is in the range of 7.86–11.38%. The order of error in current study is comparable with those from previous studies [26,27] which ranges from 8.8% to 15%.

### 4.5. Performance comparison with other EM calibrators

Performance of existing EM calibrator has been compared with other EM calibration systems reported previously, as summarized in Table 2. Our voice coil-based calibration system exhibits a comparable capability in generating both steady state and impulsive calibration forces. The calibration system has extended the steady state calibration force to four (4) orders (from 10 s to 10000 s of micronewton) of magnitude. This is attributed to the use of two magnets of different magnetic strengths in the system. The extended range of calibration force is advantageous for the torsional thrust stand to characterize the performance of a wider range of micropropulsion systems.

Apart from extended range of forces, existing system is more compact than previous EM calibration systems [22,23]. The coin-sized magnets used in this study weigh less than 10 g, which can be considered as insignificant to affect the dynamics of torsional arm. Too much additional weight mounted to the arm can cause balance and sensitivity issues. The use of voice coils has reduced the overall footprint of the system. In addition, the double-layer coiling architecture of voice coil has facilitated the production of a more uniform and stronger magnetic field.

### 5. Conclusion

In conclusion, the proposed EM calibrator has been demonstrated as a feasible alternative thrust stand calibration technique. It offers the advantages of easy to set up, requires no sophisticated equipment, cost effective and compact in size. With the combination of commercially available voice coils and various small magnets, steady-state calibration force over four orders of magnitude.
ranging from 30 to 23,000 μN was produced. Using a pulsed signal, impulsive calibration force in the range of 12–668 μNs was achieved. The system exhibits good repeatability and linearity with a maximum uncertainty of 18.47% and 11.38% for steady-state and impulsive calibrations, respectively. Installing this EM calibrator system to a torsional thrust stand provides a diagnostic tool that is capable to evaluate the performance of a variety of micropulsion systems.

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