



## Research paper

## Data mining framework for breast lesion classification in shear wave ultrasound: A hybrid feature paradigm



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## ABSTRACT

Assessment of elasticity parameters of breast using ultrasound elastography (USE) provides exclusive information about the cancerous tissue. Shear wave elastography (SWE), a new USE imaging procedure is increasingly used for elasticity evaluation of breast lesions. SWE examination is gaining popularity in the characterization of benign and malignant breast lesions as it has high diagnostic performance accuracy. However, some degree of manual errors, such as probe compression or movement may cause inaccurate results. In addition, the systems cannot measure elasticity values in small lesions where the tissues do not vibrate enough. Thus, computer-aided methods suppress these technical or manual limitations of SWE during evaluation of breast lesions. Therefore, this paper proposes, a novel methodology for characterization of benign and malignant breast lesions using SWE. Original SWE image is subjected to *three* levels of Discrete wavelet transform (DWT) to obtain different coefficients. Second order statistics (Run Length Statistics) and Hu's moments features are extracted from DWT coefficients. Extracted features are subjected to sequential forward selection (SFS) method to obtain the significant features and ranked using ReliefF feature ranking technique. Ranked features are fed to different classifiers for automated characterization of benign and malignant breast lesions. Our proposed technique achieved a significant accuracy of 93.59%, sensitivity of 90.41% and specificity of 96.39% using only *three* features. In addition, a unique integrated index named **Shear Wave Breast Cancer Risk Index (sBCRI)** is formulated for characterization of malignant and benign breast lesion using only *two* features. The proposed index, *sBCRI*, provides a single number which characterizes the malignant and benign cancer faster. This system can be employed as an ideal screening tool as it has high sensitivity and low false-positive rate. Hence, the women with benign lesions need not undergo unnecessary biopsies.

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## 1. Introduction

Breast cancer is one of the leading cancers (malignancies) in women and the second primary cause of cancer-associated deaths [11,56,119]. According to World Health Organization (WHO), in 2011, more than 508,000 breast cancer deaths are reported in women worldwide [112]. It is reported that, 1.8 million women are diagnosed with breast cancer worldwide in 2013 [48] and it is projected that in 2016, in the United States of America (USA) alone,

approximately 246,660 new cases of invasive breast cancer will be diagnosed among women [23].

Breast cancer is the result of uncontrolled multiplication of cancer cells in the breast and commonly it begins at the lobes of mammary glands [1]. This group of rapidly dividing cancer cells in the breast may eventually form a lump known as breast tumor. Breast lesion can be benign (not cancerous) or malignant (cancerous) depending on their characteristics and the degree of risk they carry [1]. Imaging of breast lesions is now regarded as essential in addition to the clinical diagnosis of breast malignancies [122]. Mammography and ultrasound (US) remain as the standard front line techniques for both screening and symptomatic disease evaluation. However, mammography performed on the dense breast may frequently produce false-negative results [22,68,69,101] and

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thus delay the diagnosis of cancer. Few studies have demonstrated that adding an US scan to screen women having the dense breast tissue together with a mammography can detect additional cancers per 1000 women [20,55]. Even though the US is highly sensitive [106,126] when applied to dense breast, lack specificity [19,31] resulting in an alarming number of false-positive; thus increasing the rate of unnecessary biopsies [49,101]. Report suggests that, using conventional US, it is difficult to precisely distinguish isoechoic lesions from the surrounding fat [89]. Therefore, despite of having regular screening tests, the breast cancers fail to be diagnosed at an early curable stage.

Clinically, it is essential to evaluate the lesion location surrounding tissue and the lesion characteristics such as size and shape according to the Breast Imaging-Reporting and Data System (BI-RADS) criteria using either mammography or an US for characterization of malignancies [87]. The primary characteristics of breast lesions include the clarity and contour of the lesion margins, the orientation and shape of the lesion, boundary echoes, the echo texture and echogenicity [17,61,106,109]. In addition, the lesion compressibility and vascularity may also be evaluated. Benign breast lesions appear round, oval minimally lobulated in shape with intense boundary echoes and homogenous internal echoes [8,18,106]. While the malignant breast lesions appear without proper margin or boundary, and may exhibit heterogeneous echo patterns, and an increased anteroposterior dimension [106]. Breast US can be used to characterize these physiological and pathological characteristics of both palpable and nonpalpable breast lesions [18,106]. Despite this, it lacks fundamental and quantitative information on the tissue elastic properties as cancer tissue is harder, stiffer (less elastic) and less compressible compared to normal breast tissue [43,102]. Moreover, improved breast US imaging modalities such as US tomography [37,98] and multi-modal US tomography (ultrasound transmission tomography (UTT) and ultrasound reflection tomography (URT)) [94,125] are intensively becoming effective in assessing the breast lesion characteristics [94,98]. Although the initial results are promising, further work with larger set of breast cancer patients are required to demonstrate their diagnostic efficacy before clinical application [94,98].

Currently, to conquer these limitations and attain more precise lesion characterization, breast US elastography (USE) is introduced [13,15,35,54,99]. Breast USE technique is used in breast lesion assessment and characterization [60]. Numerous works testified that it can improve the B-mode US specificity in distinguishing the malignant and benign breast lesions [27,28,60]. It is used to assess the tissue deformity (elasticity) and quantify the stiffness of the tissue [60,93,103].

Among the two available USE techniques such as strain elastography (SE) and shear wave elastography (SWE) [116], SWE is the only one that is highly reproducible [32,40,50,88]. In SWE, transversely aligned shear waves are produced by an acoustic radiation force during the application of an ultrasound probe to the tissue [77]. The waves travel faster in stiff (hard) tissues than in soft tissues [49]. For every pixel in the region of interest (ROI), it provides an image with color-code presenting the shear wave elasticity (kilopascals, kPa) or velocity (m/sec) [115]. Commonly, a color scale of 0 (dark blue) is used for the soft breast lesions and +180 kPa (red) for hard lesions [77]. In addition to the qualitative parameters (lesion and an adjacent tissue stiffness, lesion size, shape, and rim stiffness), quantitative parameters of the lesion such as mean elasticity (E<sub>mean</sub>), maximum and minimum elasticity (E<sub>max</sub>, E<sub>min</sub>) and elasticity ratio (E<sub>ratio</sub>) can be assessed using the SWE and are used in classification of benign and malignant breast lesions [10,67].

According to a study, elasticity parameter measured in the breast lesions (benign lesions <80 kPa and malignant lesions >100 kPa) can be a good indicator for differentiation of benign and malignant lesions [108]. Various cutoff values for the elastic-

ity parameters such as 80 kPa [24], 30 kPa [78] and 65 kPa [21] are proposed by different studies [21,25,39]. Olgun et al. [91] assessed the minimum, mean and maximum elasticity values with different cutoff in differentiating the malignant from benign lesions using a SWE. The study results showed the sensitivity and specificity of (i) 96% and 95% for the mean elasticity with a cutoff value of 45.7 kPa, (ii) 95% and 94% for the maximum elasticity with a cutoff value of 54.3 kPa, and (iii) 96% and 95% for the minimum elasticity with a cutoff value of 37.1 kPa respectively. In general, the velocity is higher for the stiffer tissue in order for the waves to travel through them. The simple lumps produce 0 velocity, because of the presence of non-viscous fluids in which the shear waves do not travel [16]. Thus, using the SWE, the degree of tissue deformation is evaluated where the stiffer tissues infiltrated with cancer deform less and stiffer (less elastic), thereby can be easily differentiated from the normal and benign surrounding tissues [71]. Various research studies are conducted for evaluating these quantitative SWE parameters in order to differentiate the malignant breast lesions from benign breast masses [21,25,40,52,66,75,76,110,114,29]. Table 1 summarizes a few of the studies on breast lesion identification using the SWE image parameters. In most of the studies, the SWE parameters are significantly higher for malignant breast lesions than the benign cases [21,25,39,76,110,123]. The mean stiffness elasticity has been found to be a useful parameter in yielding an accurate benign and malignant differentiation of solid masses [40]. In addition, for invasive breast cancers, it is shown that the breast lesions are stiffer compared to the normal lesions [41] and often produce areas of stiffness that are larger than the gray-scale abnormality generated by B-mode ultrasound [60].

However, often during the breast SWE assessments, difficulties are faced in inferring the significance of elasticity values, due to the various scanning angles providing different elasticity values. The reason for these difficulties is that breast lesions are heterogeneous and three dimensional structures [44]. To resolve this issue, Kim et al. [64] conducted a study for evaluating the effect of scanning angle on the diagnostic performance of the SWE in discriminating the breast malignancy from benign lesions. Results propose that the use of two orthogonal views that capture the images will increase the diagnostic performance of breast SWE.

Many recent studies suggest that the SWE improved the diagnostic performance accuracy and specificity of conventional US alone in the diagnosis of breast lesions [21], [26,40,75,76,82,110,113,124]. By adding SWE, about 90% of 4a masses are downgraded to BI-RADS category 3, thus, unnecessary biopsies on the benign lesions can be reduced [12,25,39,80,117]. However, it is observed that the small breast cancers are not as stiff as the larger cancers, indicating that the tumor size as well as the specific histological type can also affect the stiffness value [44,76]. In contrast, it has been reported that the diagnostic performance of elastography is better than the conventional US in the characterization of small masses (1 cm) [86]. Therefore, in 2015, Kim et al. [65] evaluated the diagnostic performance of SWE features combined with an US in the assessment of small ( $\leq 2$  cm) lesions. The study reported that by combining the two techniques, specificity increased and the number of unnecessary biopsies can be reduced while evaluating the small breast lesions. It is also shown that the breast lesion stiffness quantitatively measured by the SWE is a helpful predictor of under-estimated malignancy in an US-guided 14-gauge core needle biopsy (CNB) [95]. Lee et al. [79] claimed that the SWE is highly sensitive in an accurate identification of the presence of residual breast malignant lesions even after a neoadjuvant chemotherapy (NAC) and showed an improved diagnostic performance (sensitivity 83.6% and specificity 80%) compared to the B-mode US (sensitivity 72.1% and specificity 50%).

Recently in 2016, Ng et al. [90] investigated the efficiency of the SWE in classifying benign and malignant using 159 SWE breast

**Table 1**  
Summary of studies on breast lesion identification using the SWE image parameters.

| Author (year)     | Data  | Methods/Features   | Classification                     | Findings/Results  |
|-------------------|---|--|------------------------------------|---|
| Evans et al. [40] | Ultrasound with SWE<br>Subjects: 52 patients (23 benign and 30 malignant lesions).  | Emean  | Statistical Analysis               | Accuracy = 91%<br>Sensitivity = 97%<br>Specificity = 83%  |
| Chang et al. [25] | Ultrasound and SWE<br>Subjects: 158 women with 182 breast lesions (89 malignant, 93 benign)                                       | Emean  | Statistical Analysis               | Cutoff 80.17kPa:<br>Accuracy = 86.8%<br>Sensitivity = 88.8%<br>Specificity = 84.9%  |
| Evans et al. [39] | BI-RADS ultrasound and SWE<br>Subjects: 173 women with 175 breast lesions (64 benign and 111 malignant)                           | Emean, Emax  | Statistical Analysis               | SWE alone:<br>Accuracy = 89%<br>Sensitivity = 95%<br>Specificity = 77%<br>SWE and BI-RADS US combined:<br>Accuracy = 86%<br>Sensitivity = 100%<br>Specificity = 61%   |
| Chang et al. [24] | Ultrasound, SWE and strain Elastography<br>150 breast lesions (71 malignant, 79 benign)   | Emean and SD   | Statistical analysis and ROC       | Ultrasound:<br>Sensitivity = 100%<br>Specificity = 19%<br>SWE:<br>Sensitivity = 95.8%<br>Specificity = 84.8%<br>Strain Elastography:<br>Sensitivity = 81.7%<br>Specificity = 93.7%  |
| Youk et al. [52]  | Ultrasound and SWE<br>Subjects: 389 breast masses (269 benign and 120 malignant)  | SWE features:<br>Eratio, Emin, Emax, Emean<br>US features:<br>Diameter and depth of lesion, distance from the nipple.<br>Emax, Emean, Eratio | Chi-square and Mann-Whitney U test | Eratio (5.14 cutoff):<br>Sensitivity = 88%<br>Specificity = 90.6%   |
| Lee et al. [76]   | Ultrasound and 2D and 3D SWE<br>Subjects: 134 women with 144 breast lesions (67 malignant)  |  | ROC statistical analysis           | 2D and 3D SWE = improved specificity of ultrasound from 29.9% to 71.4%.   |
| Youk et al. [115] | Ultrasound with SWE<br>Subjects: 123 patients with 130 breast lesions (49 malignant and 81 benign)                                | Emean, Emax, SD and wSD (SD of elasticity of the whole breast lesion) measured in kPa and m/sec  | AUC of ROC                         | AUC for the wSD = 0.964 kPa and 0.960 m/sec<br><br>Specificity of SD using kPa and m/sec = 95.1% vs 87.7%   |
| Youk et al. [116] | Ultrasound, strain elastography and SWE<br>Subjects: 79 breast lesions  | Qualitative and quantitative (SWE parameters) parameters   | Statistical analysis               | SWE:<br>56% of category 4a lesions were downgraded.   |
| Lee et al. [78]   | Ultrasound and SWE<br>Development cohort: 159 breast masses (21 malignant)<br>Validation cohort: 207 breast masses (12 malignant) | Emax   | t-test                             | Reported increase in specificity.   |
| Olgun et al. [91] | Ultrasound and SWE<br>Subjects: 109 patients with 115 lesions (83 benign and 32 malignant)  | Emin, Emean, Emax, mass/fat Eratio   | Statistical Analysis - SPSS        | Emean (45.7kPa cutoff):<br>sensitivity = 96%, specificity = 95%<br>Emax (54.3 kPa cutoff):<br>sensitivity = 95%,<br>Specificity = 94%<br>Emin (37.1kPa cutoff):<br>Sensitivity = 96%<br>Specificity = 95%<br>Mass/fat Eratio:<br>Sensitivity = 97%<br>Specificity = 95% |
| Au et al. [12]    | Ultrasound and SWE<br>Subjects: 112 women with 123 masses (79 benign and 44 malignant)  | Emean, Emax, Eratio  | Statistical Analysis -SPSS         | Eratio (3.56 cutoff)<br>Combined ultrasound and SWE:<br>Accuracy = 90.24%<br>Specificity = 87.34%<br>By adding SWE parameters to BI-RADS category 4a masses, about 90% of them could be correctly downgraded to category 3, thereby avoiding biopsy.                    |
| Kim et al. [65]   | Ultrasound with SWE<br>Subjects: 171 patients with 177 small breast lesions (22 malignant and 155 benign)                         | SWE parameters – Emean, Emax, Eratio   | Statistical analysis – SPSS        | Emax (cutoff 87.5 kPa)<br>Sensitivity = 68.2%<br>Specificity = 87.1%  |

Table 1 (Continued)

| Author (year)          | Data   | Methods/Features  | Classification                                   | Findings/Results  |
|------------------------|--|---|--|---|
| Lee et al. [79]        | Ultrasound with SWE and MRI data<br>Subjects: 71 patients with breast cancers                      | E <sub>max</sub>  | ROC  | Ultrasound alone:<br>Accuracy = 69%<br>Sensitivity = 72.1%<br>Specificity = 50%<br>SWE alone:<br>Accuracy = 83.1%<br>Sensitivity = 83.6%<br>Specificity = 80%<br>Ultrasound and SWE:<br>ROC = significantly high than that of ultrasound alone  |
| Lee et al. [80]        | Ultrasound and SWE<br>Subjects: 139 patients with 140 breast lesions (30 malignant)                | SWE parameters  | Statistical Analysis                             | E <sub>max</sub> (cutoff 108.5kPa)<br>Sensitivity = 86.7%<br>Specificity = 97.3%  |
| Xian-Quan et al. [113] | Ultrasound and SWE<br>Subjects: 302 breast lesions   | E <sub>max</sub> , E <sub>mean</sub> , E <sub>min</sub> , E <sub>sd</sub> , E <sub>ratio</sub>  | Statistical Analysis                             | SWE: E <sub>max</sub> :<br>Sensitivity = 87%<br>Specificity = 97%   |
| Ng et al. [90]         | Ultrasound with SWE<br><br>Subjects: 159 breast lesions (85 benign, 74 malignant)                  | SWE parameters – Maximum elasticity (E <sub>max</sub> ), Mean elasticity (E <sub>mean</sub> ), minimum elasticity (E <sub>min</sub> ), Ratio of lesion elasticity to surrounding tissue (E <sub>ratio</sub> ), SD | Statistical analysis – SPSS                      | Sensitivity = 100%<br>Specificity = 97.6% using E <sub>max</sub> parameter in detecting malignant lesions   |
| Choi et al. [29]       | Ultrasound and SWE<br>Subjects: 81 non-mass lesions (74 malignant and 7 benign)                    | SWE:<br>E <sub>max</sub> , E <sub>mean</sub> and maximum stiffness color<br>US: Lesion size   | t-test<br>Chi-square test<br>Fisher's exact test | SWE:<br>E <sub>mean</sub> (85.1kPa cutoff)<br>Accuracy = 84.5%<br>Sensitivity = 78.4%<br>Specificity = 95.2%<br>E <sub>max</sub> (92.5kPa)<br>Accuracy = 83.6%<br>Sensitivity = 78.4%<br>Specificity = 92.9%<br>US + SWE:<br>E <sub>mean</sub> (85.1kPa or high vascularity):<br>Accuracy = 84.5%<br>Sensitivity = 95.9%<br>Specificity = 64.3% |
| Li et al. [82]         | Ultrasound and SWE<br>Subjects: 276 patients with 296 breast lesions (212 benign and 84 malignant) | SW velocity   | Statistical Analysis                             | SWE alone:<br>Sensitivity = 67.9%<br>Specificity = 86.3%  |
| Lo et al. [83]         | US and SWE<br>Subjects: 57 benign and 31 malignant tumors  | Computer-Aided Diagnostic System<br>18 SWE features (mean, variance, skewness, kurtosis, density, average from RGB color patterns)  | Logistic regression classifier                   | SWE:<br>Accuracy = 81%<br>Sensitivity = 61%<br>Specificity = 91%  |
| Zhang et al. [121]     | Ultrasound and SWE<br>Subjects: 125 women with 161 breast tumors                                   | Contourlet-based texture features (T <sub>mean</sub> , T <sub>max</sub> , T <sub>median</sub> , T <sub>qt</sub> – third quartile, T <sub>sd</sub> – standard deviation of the sub bands) – first order statistics | ROC and Fisher classifier                        | Accuracy = 92.5%<br>Sensitivity = 89.1%<br>Specificity = 94.3%  |

lesions images. The experimental results presented 100% sensitivity and 97.6% specificity with a cutoff value of  $\geq 56$  kPa for the maximum stiffness in detecting malignant lesions. In addition, their experiment claimed that, if the maximum stiffness cutoff value is kept  $\geq 80$  kPa, 95.5% of BI-RADS 4a lesions can be downgraded to BI-RADS 3, thereby negating the need for the biopsy. Even though, the SWE technique is classically less operator-dependent, some degree of variability may occur if too much pressure is applied to the probe [13]. Thus, the reliability of the SWE method depends on the operator's training and experience, if neglected, technical errors such as probe movement or compression can cause inaccurate results [32,52,88]. Moreover, manual evaluation of the SWE parameters is time-consuming and prone to errors [13]. Therefore, in order to overcome these manual or technical errors, the computer-aided methods are required for the benign and malignant breast SWE image characterization.

In 2015, Lo et al. [83], proposed a computer-aided diagnosis (CAD) tool to assess the breast lesions using SWE images. First order

statistics based features such as mean, variance, kurtosis, skewness and density of the different color channels (Red, Green, and Blue – RGB) are extracted from the tumor ROIs of the SWE. Furthermore, these features from the 3 color channels are merged as a vector to assess the tissue elasticity. The study reported that the SWE parameters showed an accuracy of 81% in classification of BI-RADS 2, 3, 4, 5 breast lesions and 83% in classification of only BI-RADS 4 category breast lesions using a logistic regression classifier. Zhang et al. [121] assessed the elastic heterogeneity of breast tumors in the SWE using contourlet-based texture analysis. They extracted the first order statistical features (T<sub>mean</sub>, T<sub>maximum</sub>, T<sub>median</sub>, T<sub>third quartile</sub> and T<sub>standard deviation</sub>) from the directional sub bands after the contourlet transform. Their study reported 92.5% accuracy, 89.1% sensitivity and 94.3% specificity in the classification of benign and malignant breast tumors. However, first order statistical features cannot capture the higher-order interrelationships present in the images. Therefore, there is a need for second-order or higher-

order and/or nonlinear feature extraction methods to capture the subtle changes occurring within the images [85].

It is evident from the literature review (Table 1) that few researchers [83,121] have proposed an automated benign and malignant breast lesion classification system using the SWE images. In view of this, in this work, a unique algorithm for an automated characterization of the benign and malignant breast lesions using SWE images is proposed. Fig. 1 shows the block diagram of our proposed algorithm. Novelty of our proposed system is the formulation of a **Shear Wave Breast Cancer Risk Index (sBCRI)** for the classification of benign and malignant breast lesions. Initially, the Discrete Wavelet Transform (DWT) is performed on the SWE images up to *three* levels to obtain different coefficients. From these DWT coefficients, various second order statistics using Run Length Statistics (RLS) and Hu's moments are extracted and subjected to sequential forward selection (SFS) method. The significant features obtained are ranked using ReliefF algorithm. All the ranked features are subjected to decision tree (DT), K nearest neighbor (KNN), linear discriminant analysis (LDA), quadratic discriminant analysis (QDA), support vector machine (SVM), and probabilistic neural network (PNN) classifiers for the automated characterization of benign and malignant. The performance of these classifiers are tested using 10-fold cross validation method.

## 2. Data collection

The data for the current study were obtained from the Department of Biomedical Imaging, University of Malaya Medical Centre (UMMC), Malaysia. The necessary institutional medical ethics committee board approval was obtained for this study. The patients were recruited from June 2012 to April 2013 by obtaining the informed consent before their recruitment. All the patients recruited were having breast lesions of BIRADS 4 category and above, and were even scheduled for ultrasound core biopsy. A total of 156 patients (73 with malignant and 83 with benign) were recruited with either palpable lumps or sonographically detected lesions. All scans were performed using the Aixplorer ultrasound system (SuperSonic Imagine, Aixen Provence, France) using a 15–4 MHz linear transducer probe. Initially, B-mode gray scale images were captured using ultrasound alone and about 3–5 min later, elastography images were generated using SWE method [90].

## 3. Methodology

### 3.1. Pre-processing

The images are cropped (about  $195 \times 195$  resolution) to have only the SWE  $3 \times 3$  cm color map in the transverse plane placed on the region encompassing the lesions and its immediate surrounding tissues. Later, the red-green-blue (RGB) color SWE images are converted into grayscale and the contrast of these images are enhanced by using adaptive histogram equalization [51].

### 3.2. Feature extraction

This stage is essential for the interpretation of abnormal and normal classes. The second order statistics RLS and Hu's moments techniques are implemented in this work to analyze the SWE images. Various RLS and Hu's moments features are evaluated from the DWT coefficients of SWE breast images. The detailed proposed methodology is explained in this section.

#### 3.2.1. Discrete wavelet transform (DWT)

This technique converts the time domain signal into wavelet domain to obtain the time and frequency values in terms of coefficients [9,92]. Filter banks are used to decompose the images into high-pass (detail) and low-pass (approximation) components [84].

By decomposing the original image using high- and low-pass filters, *four* sub band images *LL*, *LH*, *HL*, and *HH* are obtained. The *LL* coefficients signify the total energy in an image and are called approximation coefficients. The detail coefficients *HL*, *LH*, and *HH* are vertical, horizontal, and diagonal details of the image respectively. The approximation coefficients are further decomposed to obtain the second-level sub band image. These individual sub bands represented as *HDCL<sub>i</sub>*, *VDCL<sub>i</sub>*, *DDCL<sub>i</sub>*, and *ADCL<sub>i</sub>* are the horizontal, vertical, diagonal and approximation coefficients acquired at the *i*<sup>th</sup> level decomposition respectively [34]. In this work, DWT is performed on the SWE of benign and malignant images up to *three* levels using biorthogonal 3.1 (bior3.1) mother wavelet function [2]. Fig. 2 shows the *three* level DWT coefficients of benign and malignant SWE images.

It can be seen from Fig. 2 that, there is distinct variation in the pixels of benign and malignant DWT sub bands. The sub bands of malignant lesion exhibit more sudden variations as compared to benign may be due to the sudden changes in the pixels. We have extracted different features from these DWT coefficients of various sub bands.

#### 3.2.2 Gy level run-length matrix (GLRLM) or run-length statistics (RLS)

Galloway et al. [42] introduced the idea of RLS to capture the details of an image from its gray level runs. Set of image points with similar gray levels are represented by gray level run. In that run, the count of image points is a length of the run. A run-length matrix  $p(i, j)$  stores information of the count of runs with pixels of gray level *i* and run length *j*. With *M* representing the number of gray levels, *N* the number of runs, *N<sub>r</sub>* the number of different run lengths, and *N<sub>g</sub>* the number of gray levels and *P* the number of pixels in the image respectively, the Short runs emphasis ( $sR_E^x$ ), Gray level non-uniformity ( $GL_{-u}^x$ ), Long runs emphasis ( $LR_E^x$ ), Run length non-uniformity ( $RL_{-u}^x$ ) and Run percentage ( $R_p^x$ ) features are extracted using RLS methods.

Chu et al. [30] proposed a new technique based on the gray value distribution of the runs and defined two different features Low gray-level run emphasis ( $lGLR_E^x$ ) and High gray-level run emphasis ( $hGLR_E^x$ ). Dasarathy et al. [33] proposed a new method for the image characterization based on the idea of joint gray level and run length distributions. The four features Short run low gray-level emphasis ( $sRlGL_E^x$ ), Short run high gray-level emphasis ( $sRhGL_E^x$ ), Long run high gray-level emphasis ( $lRhGL_E^x$ ) and Long run low gray-level emphasis ( $lRlGL_E^x$ ) proposed by this method are extracted.

#### 3.2.3. Hu's moments ( $Hu_m^x$ )

In this work, seven moments ( $Hu_m^x$ , where  $m = 1, 2, 3, \dots, 7$ ) developed by Hu which are invariant to rotation, scaling and translation variations of the images are extracted [3], [57]. These moments capture the details about intensity distribution, geometrical (shape) features and helps to identify the patterns in images [57], [120].

### 3.3. Feature selection and ranking

All nonlinear features assessed are not capable of classifying the benign and malignant groups. Thus, to recognize the extremely resourceful features with useful information, sequential forward feature selection method is used. The Sequential forward selection (SFS) selects the features sequentially and adds to an empty sub set until the addition of features maximizes the classification accu-

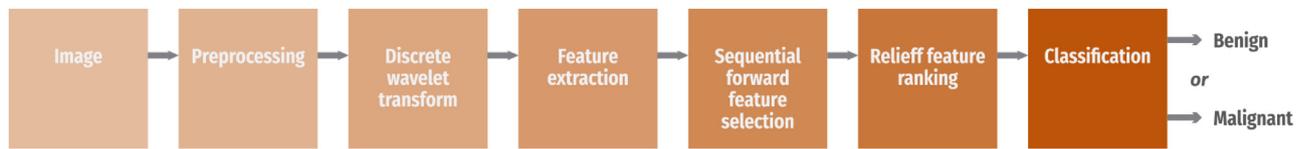


Fig. 1. Block diagram of proposed algorithm.

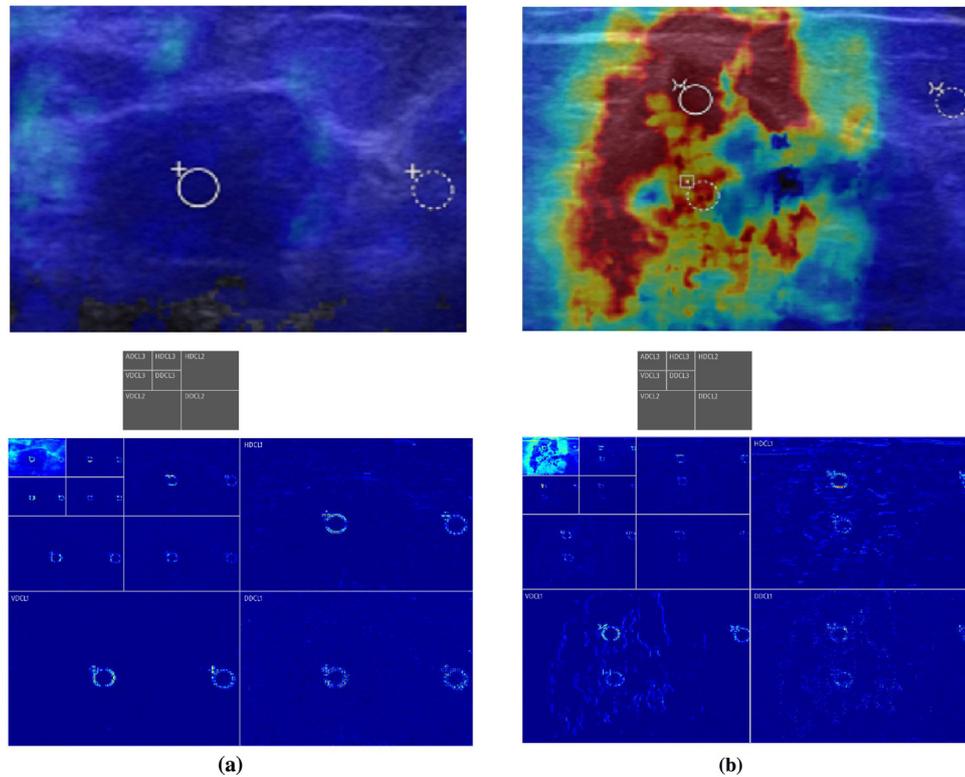


Fig. 2. Three-level sub-bands of 2D DWT (bottom row) of SWE images (top row): (a) benign and (b) malignant.

acy [70], [81]. The main advantages of using this SFS method is its computational efficiency and avoids over fit [72]. In addition, this SFS method of feature selection has frequently shown to perform competitively compared to other method like sequential floating forward selection (SFFS) [96], [97]. Thus, this method is considered as one of the state-of-the-art feature selection techniques. Selected features are subjected to ReliefF feature ranking method which uses the  $k$  nearest neighbors – hits (from the same class) and misses (from the different class). The averages of their contributions are added to Relief's estimate [100].

### 3.4. Classification

Classification is performed on the ranked features using different classifiers in order to attain the maximum outcome. In this work, DT, LDA, QDA, SVM Polynomial 1, 2, 3 and RBF, PNN and KNN classifiers are experimented to find the best classifier in distinguishing the benign and malignant SWE images [36]. **DT** classifier, builds a tree from the features during the training phase [73]. Based on the rules obtained from the tree it built, classifies the two classes and thereby identifies the class of test (unknown) data. **KNN** classifier determines  $k$ -nearest neighbors i.e. the minimum distance between training and testing data [74]. The unknown sample gets the most common class among the  $k$ -nearest neighbors. **SVM** classifier in the higher-dimensional space constructs a separating hyperplane which splits the training sets into two classes [38]. In

this work, we have used polynomial 1, 2, 3 and RBF kernel functions. **PNN** classifier works on the principle of supervised learning algorithm and calculates the weights. Using a multi-layered neural network having 4 layers, this classifier is often employed in classification of two groups [111]. Two types of **discriminant classifiers** such as LDA and QDA are employed in this work. LDA and QDA learns from linear and quadratic boundaries respectively [59], [63]. A 10-fold cross validation technique is employed in our work for performance validation of the classifiers.

## 4. Results

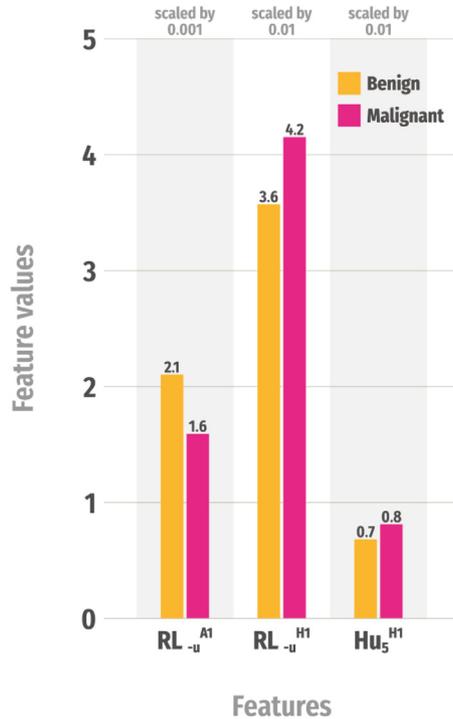
### 4.1. Results of feature extraction

The results of feature extraction and classification are presented in this section. Each SWE image is subjected to *three* level DWT to obtain a total of 12 DWT coefficients. Total of 216 (12 DWT coefficients  $\times$  (11 RLS + 7 Hu's moments)) features are computed. All together  $216 \times 156$  features are computed from both benign and malignant breast lesions.

Table 2 shows the *mean* and standard deviation (SD) values of second order (RLS) and nonlinear (Hu's moments) features extracted from the DWT coefficients of benign and malignant SWE images. Significant features selected using SFS method are ranked by ReliefF technique and the best 3 highly ranked features are tabulated in Table 2 and Fig. 3.

**Table 2**  
Results (mean  $\pm$  SD) of top three features extracted from benign and malignant breast SWE images using RLS and Hu's moments methods.

| Features       | Benign   |          | Malignant |          | p-Value | Criterion |
|----------------|----------|----------|-----------|----------|---------|-----------|
|                | Mean     | SD       | Mean      | SD       |         |           |
| $RL_{-u}^{A1}$ | 2.10E+03 | 1.78E+02 | 1.59E+03  | 2.35E+02 | 0.0000  | 0.0229    |
| $RL_{-u}^{H1}$ | 3.57E+02 | 1.16E+02 | 4.15E+02  | 1.46E+02 | 0.0067  | 0.0083    |
| $Hu_5^{H1}$    | 6.81E+01 | 2.16E+01 | 8.08E+01  | 3.03E+01 | 0.0028  | 0.0048    |



**Fig. 3.** Bar plots of mean values of extracted features from benign and malignant SWE images using RLS and Hu's moments method.

It is evident from Table 2 and Fig. 3 that the feature values are higher in malignant lesions compared to the benign lesions. This is may be due to the more abrupt changes in the pixels of malignant lesion. In benign lesion, the pixel variations are less abrupt as compared to malignant contributing to the smaller values.

#### 4.2. Results of classification

Ranked features are fed to different classifiers namely, DT, LDA, QDA, SVM, KNN and PNN to achieve the highest classification performance. QDA classifier has achieved the maximum accuracy, sensitivity and specificity of 93.59%, 90.41% and 96.39% respectively using the DWT method with three features ( $RL_{-u}^{A1}$ ,  $RL_{-u}^{H1}$ ,  $Hu_5^{H1}$ ). Table 3 shows the results of classification obtained using second-order statistics, RLS and Hu's moments. It is evident from this Table 3 that the QDA outperformed other classifiers in achieving the highest results.

We have also performed classification without the DWT using only RLS and Hu's moments directly from the SWE images. We have achieved 88.46% accuracy, 83.56% sensitivity and 92.77% specificity using only two features. Therefore, extracting features (RLS and Hu's moments) from the DWT coefficients has significantly improved the classification performance. Thus, fusion of three techniques DWT, RLS and Hu's moments showed better results in an efficient characterization of the benign and malignant breast lesions from SWE images.



**Fig. 4.** Plot of sBCRI for benign and malignant breast lesions.

#### 4.3. Results of index – sBCRI

Apart from the classifiers, an idea of integrated index, first introduced by Ghista [45], [46,47], is proposed for the characterization of benign and malignant breast lesions in this work. An integrated index named sBCRI is formulated by merging the most distinctive features in such a way that the index value is noticeably different for the two classes.

In the current work, a sBCRI is formulated using the two features such as  $RL_{-u}^{A1}$  and  $Hu_5^{H1}$  obtained from the SWE images (Table 2). These two features are then combined (see Eq. (1)) to produce an index which is highly distinguishing. Mathematical equation of this sBCRI is given as,

$$sBCRI = \frac{(1.5 \times (RL_{-u}^{A1}) + 2.2 \times (Hu_5^{H1}) + 5.5)}{1000} \quad (1)$$

This sBCRI formulated gives a maximum distinction between the two classes (benign and malignant). Table 4 shows the sBCRI values for the subjects with malignant and benign breast lesions. A plot of sBCRI for the benign and malignant cases obtained using the combination of two features ( $RL_{-u}^{A1}$  and  $Hu_5^{H1}$ ) is shown in Fig. 4. Thus, sBCRI (a single number) aids the clinicians in faster and an accurate characterization of the benign and malignant breast lesions.

## 5. Discussion

In this work, a unique algorithm is proposed by fusing DWT, texture and Hu's moments features. DWT method decomposes SWE

**Table 3**  
Results of classification of benign and malignant SWE image features obtained using RLS and Hu's moments methods.

| Classifiers   | NOF | TP | TN | FP | FN | Acc (%) | Sen (%) | Spe (%) |
|---------------|-----|----|----|----|----|---------|---------|---------|
| DT            | 3   | 62 | 76 | 7  | 11 | 88.46   | 84.93   | 91.57   |
| LDA           | 3   | 64 | 81 | 2  | 9  | 92.95   | 87.67   | 97.59   |
| QDA           | 3   | 66 | 80 | 3  | 7  | 93.59   | 90.41   | 96.39   |
| SVM Poly 1    | 3   | 64 | 81 | 2  | 9  | 92.95   | 87.67   | 97.59   |
| SVM Poly 2    | 3   | 62 | 80 | 3  | 11 | 91.03   | 84.93   | 96.39   |
| SVM Poly 3    | 3   | 66 | 78 | 5  | 7  | 92.31   | 90.41   | 93.98   |
| KNN – 25      | 3   | 63 | 82 | 1  | 10 | 92.95   | 86.30   | 98.80   |
| PNN – 0.15    | 2   | 61 | 82 | 1  | 12 | 91.67   | 83.56   | 98.80   |
| SVM RBF – 1.4 | 3   | 66 | 79 | 4  | 7  | 92.95   | 90.41   | 95.18   |

Acc = accuracy; Sen = sensitivity; Spe = specificity; TP = true positive; TN = true negative; FP = false positive; FN = false negative; NOF = number of features.

**Table 4**  
Range of a *sBCRI* for the benign and malignant breast lesions.

| Benign   |         | Malignant |          |
|----------|---------|-----------|----------|
| Mean     | SD      | Mean      | SD       |
| 3.306607 | 0.27379 | 2.570718  | 0.367312 |

image into the sets of images at different positions and scales by using filter banks. Hence this time-invariant decomposition method (DWT) is very effective in separating noise from an image [62,107]. An important benefit of this method is its ability to treasure temporal resolution of the images; in other words, it captures both frequency and location details [6,84]. Furthermore, DWT technique is robust and useful in detecting irregularities present in the images [4,6,7]. Thus, DWT multiresolution analysis technique is advantageous for obtaining an efficient performance using the SWE images.

Several other researchers have analyzed the SWE images using quantitative parameters [21,25,40,64,66,76,90,110], texture features [83,121] in the characterization of benign and malignant breast lesions. Manual evaluation of breast lesion parameters using SWE is prone to errors during data acquisition and visual interpretation variabilities present in different USE equipment manufacturers [13,104]. An elasticity value sometimes cannot be calculated when the deformation of tissue is too low. This may occur in large very rigid infiltrative cancers [110]. The systems cannot measure the elasticity values in lesions where tissue does not vibrate enough or the amplitude of shear wave is too low and thus lost in the background noise [14]. Chances of having more false positive is high for large size lesion and thick breast, and more false-negative values for the smaller and deeper lesions [114].

Thus, analysis of the image texture characteristics using computer-aided technique may overcome these limitations of manual evaluation of SWE parameters. Moreover, the high quality SWE images have shown an improved diagnostic performance in characterizing the malignant breast lesions from benign ones [25,76]. Hence, there is a need for an efficient image pre-processing technique before extraction of the features.

Hence, to surpass the obstacles, a novel technique for an automated characterization of the benign and malignant breast lesions from SWE images is proposed in this work. DWT is performed on the benign and malignant SWE images. Various RLS and Hu's moments features are computed from the DWT coefficients. Significant features are selected from the extracted parameters using SFS method. All these selected features are ranked according to the Relief ranking method. Later, the ranked features are classified using different classifiers. In addition, an index called *sBCRI* is formulated and developed using the second-order RLS and nonlinear Hu's moments features extracted from the SWE images for characterization of malignant and benign breast lesions. Our proposed method showed 93.59% accuracy, 90.41% sensitivity and 96.39% specificity in characterizing the benign and malignant breast lesions using

only three ( $RL_{-u}^{A1}, RL_{-u}^{H1}, Hu_5^{H1}$ ) features extracted from the DWT coefficients (approximation and horizontal). We have also performed the classification using same features without DWT technique. Results showed low performance in the characterization of malignant and benign as compared to the DWT technique. Therefore, fusion of DWT technique exhibited the highest performance in the characterization of malignant and benign SWE images.

Moreover, the second-order and nonlinear features (RLS and Hu's moments) used in this work are capable of extracting the spatial interrelationships and complexity present in pixels of images [5,118]. Second order statistics evaluate the properties of more than one pixel values at specific positions relative to each other [105]. The second order statistics calculated using RLS method provides for each sample the large set of features, thus, even small variations (subtle changes) occurring in an image texture are captured by these features [53,105]. Hu's moments are widely used in image pattern recognition due to its invariant nature of image translation, scaling and rotation [57,58]. Moreover, these features (RLS and Hu's moments) evaluated from the DWT coefficients provide information related to the different frequency bands present in an image [5,118]. Therefore, in essence, the second-order statistics and nonlinear features are suitable for obtaining information from the SWE images and can be successfully used for the development of an efficient diagnostic system.

In comparison to the other published studies, our proposed method achieved better sensitivity and specificity in differentiating the malignant and benign SWE images. This proposed technique can assist, if used in hospitals or polyclinics to diagnose the malignancy accurately and hence avoid the unnecessary biopsies as it has high specificity. The main novelty of our proposed work is the formulation of an **integrated index sBCRI** for the characterization of malignant breast lesions using only the two features extracted from SWE images. In future work, we will be exploring to develop an automated system using SWE for the characterization of benign, pre-malignant and malignant breast lesions.

## 6. Conclusion

Automated characterization of breast malignancy from benign lesions using SWE is a challenging task. In this paper, an **integrated index, sBCRI**, for an automated characterization of breast malignant lesions is proposed. DWT technique performed on SWE images has achieved the high accuracy, sensitivity and specificity of 93.59%, 90.41% and 96.39% respectively using three features in the characterization of malignant and benign lesions. The proposed technique is reliable and promising. In addition, our system is affordable to the clinicians without the need for any special hardware setup. Thus, this software can be incorporated in the routine clinical practice as a screening tool to diagnose breast lesions.

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