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# Performance of Finite order Distribution-Generated Universal Portfolios

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**Abstract.** A Constant Rebalanced Portfolio (CRP) is an investment strategy which reinvests by redistributing wealth equally among a set of stocks. The empirical performance of the distribution-generated universal portfolio strategies are analysed experimentally concerning 10 higher volume stocks from different categories in Kuala Lumpur Stock Exchange. The time interval of study is from January 2000 to December 2015, which includes the credit crisis from September 2008 to March 2009. The performance of the finite-order universal portfolio strategies has been shown to be better than Constant Rebalanced Portfolio with some selected parameters of proposed universal portfolios.

## INTRODUCTION

The idea of using a probability distribution to generate a universal portfolio is due to Cover [1]. The Cover-Ordentlich universal portfolio [2] is a moving-order universal portfolio. This moving-order universal portfolio are not practical in the sense that as the number of stocks in the portfolio increases, the implementation time and the computer storage requirements grow exponentially fast. A finite-order universal portfolio generated by some probability distribution, due to [3] with comparable performance and requiring faster implementation time and much lesser computer memory is introduced. This type of universal portfolio depends only on the positive moments of the generating probability distribution.

**TABLE 1**. Ten most active stocks from different categories

| Category | Stock Code | Stock Name | Average Volume | Period |
|---|---|---|---|---|
| Construction | 5398 | Gamuda Bhd | 4986282.33 | January 2000 to December 2015 |
| Consumer Product | 7084 | QL Resources Berhad | 2086150 | March 2000 to December 2015 |
| Finance | 1818 | Bursa Malaysia Bhd | 51271350 | January 2005 to December 2015 |
| Hotel | 5517 | Shangri-La Hotels Malaysia Bhd | 69100 | January 2000 to December 2015 |
| Industry Products | 7106 | Supermax Corporation Bhd | 12656000 | January 2000 to December 2015 |
| IPC | 5031 | Time Dotcom Bhd | 630100 | March 2001 to January 2015 |
| Plantation | 2216 | IJM Plantation Bhd | 10168800 | July 2003 to December 2015 |
| Properties | 5148 | UEM Sunrise Bhd | 9561550 | January 2000 to December 2015 |
| Trading Services | 5099 | Air Asia Bhd | 94589600 | November 2004 to December 2015 |
| Trading Services | 3182 | Genting Bhd | 13869700 | January 2000 to December 2015 |

We present an experimental study of two finite order distribution-generated universal portfolios, namely the finite order Multinomial universal portfolio and the finite order Multivariate Normal universal portfolio. Ten most active

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stocks data from Kuala Lumpur Stock Exchange with higher volume from different categories are selected from the top 100 listed companies [4]. The day-end KLSE data was obtained from [5]. The database contains daily opening prices, daily closing prices, daily high and low, and the volume of transaction. These ten stocks data with their respective code are shown in Table 1. The trading period is between January 2000 to December 2015. The above order one universal portfolios are run on every dataset consist of three stock data generated from the combination of these 10 selected most active stocks. The wealth achieved after the $n$ trading days by the above portfolio strategies is compared to the wealth obtained by CRP strategies. The well performing parameters of the above two universal portfolio strategies are observed.

## GENERAL METHOD FOR UNIVERSAL PORTFOLIO GENERATION

Consider an $m$-stock market. Let $\mathbf{x}_n = (x_{ni})$ be the stock-price-relative vector on the $n^{th}$ trading day, where $x_{ni}$ denotes the stock-price relative of stock $i$ on day $n$, which is defined to be the ratio of the closing price to its opening price on day $n$, for $i = 1, 2, \cdots, m$. Let $\hat{\mathbf{b}}_n = (\hat{b}_{n,i})$ denotes the universal portfolio vector on the $n^{th}$ trading day, where $\hat{b}_{ni}$ is the proportion of the current wealth on day $n$ invested on stock $i$, for $i = 1, 2, \cdots, m$ and $\sum_{i=1}^{m} b_{ni} = 1$. The initial wealth $\hat{S}_0$ is assumed to be one unit and the wealth at the end of $n^{th}$ trading day $\hat{S}_n$ is giving by

$$\hat{S}_n = \hat{\mathbf{b}}_1^t \mathbf{x}_1 \times \hat{\mathbf{b}}_2^t \mathbf{x}_2 \times \cdots \times \hat{\mathbf{b}}_n^t \mathbf{x}_n \tag{1}$$

where $\mathbf{b}^t$ denotes the transpose of the vector $\mathbf{b}$.

The theory of universal portfolio order $\nu$ generated by probability distribution is due to [3]. Let $Y_1, Y_2, \cdots, Y_m$ be $m$ discrete/continuous random variables having joint probability mass/density function $f(y_1, \cdots, y_m)$ defined over the domain $D$ where

$$D = \{(y_1, \cdots, y_m) : f(y_1, \cdots, y_m) > 0\}. \tag{2}$$

Furthermore, let $\mathbf{y}$ denote the vector $(y_1, \cdots, y_m)$. If $Y_1, Y_2, \cdots, Y_m$ are mutually independent, we can have a mixture of discrete and continuous random variables. Given a positive integer $\nu$, the universal portfolio $\hat{\mathbf{b}}_{n+1}$ of order $\nu$ generated by $Y_1, Y_2, \cdots, Y_m$ is defined as

$$\hat{b}_{n+1,k} = \frac{\int_D y_k (\mathbf{y}^t \mathbf{x}_n)(\mathbf{y}^t \mathbf{x}_{n-1}) \cdots (\mathbf{y}^t \mathbf{x}_{n-(\nu-1)} f(y_1, \cdots, y_m) d\mathbf{y}}{\int_D (y_1 + \cdots + y_m)(\mathbf{y}^t \mathbf{x}_n)(\mathbf{y}^t \mathbf{x}_{n-1}) \cdots (\mathbf{y}^t \mathbf{x}_{n-(\nu-1)} f(y_1, \cdots, y_m) d\mathbf{y}} \tag{3}$$

## FINITE ORDER DISTRIBUTION-GENERATED UNIVERSAL PORTFOLIOS AND CONSTANT REBALANCED PORTFOLIO

### The Finite Order Multinomial Universal Portfolio

Let the random variables $Y_1, Y_2, \cdots, Y_m$ have a joint multinomial distribution with parameters $N, p_1, p_2, \cdots, p_{m-1}$ where $0 < p_i < 1$ for $i = 1, 2, \cdots, m-1$ and $0 < p_m = 1 - p_1 - p_2 - \cdots - p_{m-1} < 1$; $N$ is a positive integer bigger than $m$, which is the number of stocks in the market. The joint probability function $f(y_1, y_2, \cdots, y_m)$ is given by

$$f(y_1, y_2, \cdots y_m) = \begin{pmatrix} & N & & \\ y_1 & y_2 & \cdots & y_m \end{pmatrix} p_1^{y_1} p_2^{y_2} \cdots p_m^{y_m} \tag{4}$$

where the multinomial coefficient $\begin{pmatrix} & N & & \\ y_1 & y_2 & \cdots\cdot & y_m \end{pmatrix}$ is defined by

$$\begin{pmatrix} & N & & \\ y_1 & y_2 & \cdots & y_m \end{pmatrix} = \frac{N!}{y_1! y_2! \cdots y_m!} \tag{5}$$

and $y_i = 0, 1, 2, \cdots, N$ for $i = 1, 2, ..., m$ subject to $y_1 + y_2 + \cdots + y_m = N$. The joint moment generating function $M(\tau_1, \tau_2 \cdots, \tau_m)$ of the multinomial distribution (4) is given by

$$M(\tau_1, \tau_2, \cdots, \tau_m) = (p_1 e^{\tau_1} + p_2 e^{\tau_2} + \cdots + p_m e^{\tau_m})^N. \tag{6}$$

Let $M^{n_1,n_2,\cdots,n_m}(\tau_1,\tau_2,\cdots,\tau_m)$ denote the partial derivative $\frac{\partial^{n_1+n_2+\cdots n_m} M(\tau_1,\tau_2,\cdots,\tau_m)}{\partial\tau_1^{n_1}\partial\tau_1^{n_2}\cdots\partial\tau_m^{n_m}}$ and $M^{n_1,n_2,\cdots,n_m}(0,0,...,0)$ will denote the value of the partial derivative evaluated at $\tau_1=\tau_2=\cdots=\tau_m=0$. It is known that

$$E[Y_1^{n_1}Y_2^{n_2}.\cdots Y_m^{n_m}]=M^{n_1,n_2,.n_m}(0,0,\cdots,0) \tag{7}$$

If $n_k$ in (7) is increased by 1, then we adopt the notation

$$E[Y_1^{n_1\cdots}Y_k^{n_k+1}\cdots Y_m^{n_m}]=M^{n_1,n_2,\cdots,n_k+1,\cdots n_m}(0,0,\cdots,0), \quad \text{for } k=1,2,\cdots,m. \tag{8}$$

Let $\mathbf{x}_n=(x_{ni})\geq 0$ be the nonnegative price-relative or asset-relative vector on the $n^{th}$ trading day for $n=1,2,\cdots$, where the price-relative of a stock/asset is defined to be the ratio of the closing price to its opening price [2]. We adopt the notation $\mathbf{x}^n=(\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_n)$ to denote a sequence of price-relative vectors from first to the $n^{th}$ trading day.

Let the domain $D$ of the function $f(\mathbf{y})$ given by (4) be

$$D=\{\mathbf{y}: f(\mathbf{y})>0,\sum_{i=1}^{m} y_i=N, y_i=0,1,....,N for, i=1,2,\cdots,N\} \tag{9}$$

where the vector $\mathbf{y}=(y_1,y_2,\cdots,y_m)$. Given a positive integer $\nu$ and $\mathbf{x}$, the multinomial universal portfolio of order $\nu$ is defined by the sequence of portfolio vectors $\hat{\mathbf{b}}_{n+1,k}=(b_{n+1,k})$ given by

$$\hat{\mathbf{b}}_{n+1,k}=\frac{\sum_{\mathbf{y}\in D} y_k(\mathbf{y}^t\mathbf{x}_n)(\mathbf{y}^t\mathbf{x}_{n-1})...(\mathbf{y}^t\mathbf{x}_{n-(\nu-1)})f(\mathbf{y})}{N\sum_{\mathbf{y}\in D}(\mathbf{y}^t\mathbf{x}_n)(\mathbf{y}^t\mathbf{x}_{n-1})....(\mathbf{y}^t\mathbf{x}_{n-(\nu-1)})f(\mathbf{y})} \tag{10}$$

for $k=1,2,\cdots,m$ and $n=1,2\cdots,$; $f(\mathbf{y})$ and $D$ are given by (4) and (9) respectively. We shall omit $D$ whenever the domain of summation is clear in the context.Note that (10) can be rewritten in the form

$$\hat{\mathbf{b}}_{n+1,k}=\zeta_{n,\nu}\{\sum_{i_1=1}^{m}\sum_{i_2=1}^{m}\cdots\sum_{i_\nu=1}^{m}(x_{n_{i_1}}x_{n-1,i_2}\cdots x_{n-\nu+1,i_\nu})\times E[Y_1^{n_1(k;\mathbf{i})}Y_2^{n_2(k;\mathbf{i})}.\cdots Y_m^{n_m(k;\mathbf{i})})]\} \tag{11}$$

for $k=1,2,\cdots m$ where the constant $\zeta_{n,\nu}$ is given by

$$\zeta_{n,\nu}=\{N\sum_{i_1=1}^{m}\sum_{i_2=1}^{m}\cdots\sum_{i_\nu=1}^{m}(x_{n_{i_1}}x_{n-1,i_2}\cdots x_{n-\nu+1,i_\nu})\times E[Y_1^{n_1(\mathbf{i})}Y_2^{n_2(\mathbf{i})}\cdots Y_m^{n_m(\mathbf{i})}]\}^{-1}, \tag{12}$$

the vector $\mathbf{i}=(i_1,i_2,\cdots,i_\nu); 1\leq i_j\leq m$ for $j=1,2,\cdots m; n_j(k;\mathbf{i})$ is the number of $y_j^{'}$s in the product $(y_k y_{i_1} y_{i_2}\cdots y_{i_\nu})$ ; $n_j(\mathbf{i})$ is the number of $y_j^{'}$ s in the product$(y_{i_1}y_{i_2}\cdots y_{i_\nu})$; $\sum_{j=1}^{m} n_j(k;\mathbf{i})=\nu+1$ and $\sum_{j=1}^{m} n_j(\mathbf{i})=\nu$. To see this, consider the numerator of $\hat{b}_{n+1,k}$ in (10):

$$\begin{aligned}
&\sum_{\mathbf{y}} y_k(\sum_{i_1=1}^{m} y_{i_1}x_{ni_1})(\sum_{i_2=1}^{m} y_{i_2}x_{n-1,i_2})....(\sum_{i_\nu=1}^{m} y_{i_\nu}x_{n-\nu+1,i_\nu})f(\mathbf{y})\\
&=\sum_{\mathbf{y}} y_k[\sum_{i_1=1}^{m}\sum_{i_2=1}^{m}\cdots\sum_{i_\nu=1}^{m}(y_{i_1}y_{i_2}....y_{i_\nu})(x_{ni_1}x_{n-1,i_2}\cdots x_{n-\nu+1,i_\nu}]f(\mathbf{y})\\
&=\sum_{i_1=1}^{m}\sum_{i_2=1}^{m}...\sum_{i_\nu=1}^{m}(x_{ni_1}x_{n-1,i_2}\cdots x_{n-\nu+1,i_\nu}][\sum_{\mathbf{y}}(y_k y_{i_1}y_{i_2}\cdots y_{i_\nu})f(\mathbf{y})]
\end{aligned}$$

which is the same as the numerator of (11) by counting the number of $y_1'$s, $y_2'$s.....$y_m'$s in the product $(y_k y_{i_1}y_{i_2}....y_{i_m})$. A similar derivation shows that the denominators of (10) and (11) are the same.

## The Finite Order Multivariate Normal Universal Portfolio

Let $\nu$ be a positive integer. Let the random variables Y = $(Y_1,Y_2,\cdots,Y_m)$ have a joint multivariate normal probability density function $f(y_1,y_2,\cdots y_m)$ defined over $B$, where $B=\{(y_1,y_2,\cdots y_m):-\infty<y_i<\infty, i=1,\cdots,m, f(y_1,\cdots y_m)>0\}$, where

$$f(\mathbf{y}) = \frac{e^{-\frac{1}{2}(\mathbf{y}-\mu)^t K^{-1}(\mathbf{y}-\mu)}}{(\sqrt{2\pi})^n |K|^{1/2}}, \tag{13}$$

$K$ is the covariance matrix of $Y$, $\mu = (E(Y_1), E(Y_2), \cdots, E(Y_m))$ is the mean vector. We say Y has the multivariate normal distribution $N(\mu, K)$ , $|\cdot|$ means determinate.

The universal portfolio $\hat{b}_{n+1,k}$ of order $\nu$ generated by the joint p.d.f $f(\mathbf{y})$ is defined as :

$$\hat{\mathbf{b}}_{n+1,k} = \frac{\int_B y_k(\mathbf{y}^t\mathbf{x}_n)(\mathbf{y}^t\mathbf{x}_{n-1})\cdots(\mathbf{y}^t\mathbf{x}_{n-(\nu-1)})f(\mathbf{y})d(\mathbf{y})}{\int_B (y_1+\cdots+y_m)(\mathbf{y}^t\mathbf{x}_n)(\mathbf{y}^t\mathbf{x}_{n-1})\cdots(\mathbf{y}^t\mathbf{x}_{n-(\nu-1)})f(\mathbf{y})d(\mathbf{y})} \tag{14}$$

for $k = 1, 2, \cdots, m; y = (y_1, y_2, \cdots, y_m), n = 0, 1, 2, \cdots$ . The theory of universal portfolio order $\nu$ generated by probability distribution is due to [3], where we assume that $E[Y_1^{n_1} Y_2^{n_2} .....Y_m^{n_m}] \geq 0$ for all non-negative integers $n_1, n_2, \cdots, n_m$ satisfy $0 \leq n_i \leq \nu + 1$ for $i = 1, 2, \cdots, m$ and $\sum_{i=1}^m = \nu + 1$. The numerator of $\hat{b}_{n+1,k}$ can be rewritten as

$$\begin{aligned}
&\int_B y_k(\sum_{i_1=1}^m y_{i_1} x_{ni_1})(\sum_{i_2=1}^m y_{i_2} x_{n-1,i_2})(\sum_{i_\nu=1}^m (y_{i_\nu} x_{n-\nu+1,i_\nu}))f(\mathbf{y})d(\mathbf{y}) \\
&= \int_B y_k[\sum_{i_1=1}^m \sum_{i_2=1}^m \cdots \sum_{i_\nu=1}^m (y_{i_1} y_{i_2} \cdots y_{i_\nu})(x_{ni_1} x_{n-1,i_2} \cdots x_{n-\nu+1,i_\nu})]f(\mathbf{y})d(\mathbf{y}) \\
&= \sum_{i_1=1}^m \sum_{i_2=1}^m \cdots \sum_{i_\nu=1}^m (x_{ni_1} x_{n-1,i_2} \cdots x_{n-\nu+1,i_\nu}) E[Y_1^{n_1(k;\mathbf{i})} Y_2^{n_2(k;\mathbf{i})} \cdots Y_m^{n_m(k;\mathbf{i})}]
\end{aligned} \tag{15}$$

where $n_j(k;\mathbf{i})$ is the number of $y_j$'s in the product $(y_k y_{i_1} y_{i_2} \cdots y_{i_\nu})$, $i = (i_1, i_2, \cdots, i_m)$ for $1 \leq i_j \leq m$ for $j = 1, 2, \cdots, m$ ; $0 \leq n_j(k;\mathbf{i}) \leq \nu + 1$ and $\sum_{j=1}^m n_j(k;\mathbf{i}) = \nu + 1$. The denominator of $\hat{\mathbf{b}}_{n+1,k}$ is normalizing constant $\zeta_{n+1}^{-1}$. where

$$\zeta_{n+1} = \left\{ \sum_{k=1}^m [\sum_{i_1=1}^m \cdots \sum_{i_\nu=1}^m (x_{ni_1} x_{n-1,i_2} \cdots x_{n-\nu+1,i_\nu}) E[Y_1^{n_1(k;\mathbf{i})} Y_2^{n_2(k;\mathbf{i})} \cdots Y_m^{n_m(k;\mathbf{i})}]] \right\}^{-1} \tag{16}$$

Thus

$$\hat{\mathbf{b}}_{n+1,k} = \zeta_{n+1} \sum_{i_1=1}^m ..... \sum_{i_\nu=1}^m (x_{ni_1} x_{n-1,i_2} \cdots x_{n-\nu+1,i_\nu}) E[Y_1^{n_1(k;\mathbf{i})} Y_2^{n_2(k;\mathbf{i})} \cdots Y_m^{n_m(k;\mathbf{i})}] \tag{17}$$

for $k = 1, 2, \cdots, m$ . Note that $\zeta_{n+1}$ in (16) can also be written as

$$\zeta_{n+1} = \left\{ \sum_{i_1=1}^m \cdots \sum_{i_\nu=1}^m (x_{ni_1} x_{n-1,i_2} \cdots x_{n-\nu+1,i_\nu}) \times E[(Y_1 + Y_2 + \cdots + Y_m)(Y_1^{n_1(\mathbf{i})} Y_2^{n_2(\mathbf{i})} \cdots Y_m^{n_m(\mathbf{i})}] \right\}^{-1} \tag{18}$$

where $n_j(\mathbf{i})$ is the number of $y_j^{'}s$ in the sequence $y_{i_1}, y_{y_2} \cdots y_{i_\nu}$ for $j = 1, 2, \cdots, m$ ; $0 \leq n_j(\mathbf{i}) \leq \nu$ and $\sum_{j=1}^m n_j(\mathbf{i}) = \nu$.

The Multivariate normal distribution $N(\mu, k)$ is of special form $\mu = (\mu_1, \mu_2, \cdots, \mu_m)$ and $K = diag(\sigma_1, \sigma_1, \cdots, \sigma_m)$ where $Y_1, Y_2, \cdots, Y_m$ are independent.

The joint moment-generating function $M(\tau)$= $M(\tau_1, \tau_2, \cdots \tau_m)$ of the Multivariate normal distribution is given by

$$M(\tau_1, \tau_2, \cdots, \tau_m) = e^{\mu^t + \frac{1}{2}(\tau_1, \tau_2, \cdots \tau_m)^t K(\tau_1, \tau_2, \cdots, \tau_m)} \tag{19}$$

Let $M^{n_1, n_2, \cdots, n_m}(\tau_1, \tau_2, \cdots, \tau_m)$ denote the partial derivative $\frac{\partial^{n_1+n_2+\cdots n_m} M(\tau_1, \tau_2, \cdots, \tau_m)}{\partial \tau_1^{n_1} \partial \tau_1^{n_2} \cdots \partial \tau_m^{n_m}}$ and

$M^{n_1, n_2, \cdots, n_m}(0, 0, \cdots, 0)$ will denote the value of the partial derivative evaluated at $\tau_1 = \tau_2 = \cdots = \tau_m = 0$. It is known that

$$E[Y_1^{n_1} Y_2^{n_2} \cdots Y_m^{n_m}] = M^{n_1, n_2, \cdots n_m}(0, 0, \cdots, 0) \tag{20}$$

If $n_k$ in (20) is increased by 1, then we adopt the notation

$$E[Y_1^{n_1} \cdots Y_k^{n_k+1} \cdots Y_m^{n_m}] = M^{n_1, n_2, \cdots, n_k+1, \cdots n_m}(0, 0, \cdots, 0) \tag{21}$$

for $k = 1, 2, \cdots, m$.

## Constant Rebalanced Portfolio

A *constant-rebalanced portfolio* is a portfolio $\mathbf{b} = (b_i)$ that is constant over the trading days and the wealth at the end of $n$ trading days is

$$S(\mathbf{x}^n) = \prod_{i=1}^{n} \mathbf{b}^t \mathbf{x}_n. \quad (22)$$

# SELECTED PARAMETERS FOR UNIVERSAL PORTFOLIOS

The above two finite-order universal portfolios are studied on 10 most active stock-price data selected from the Kuala Lumpur Stock Exchange, which described in Introduction section. Every three stock data are generated from the 10 stocks data by using combination. In [6] and [7], good performances are obtained by order one Multinomial generated universal portfolio and order one Multivariate Normal generated universal portfolio, with the wealth achieved outperform the Diriclet universal portfolio. Therefore, only order one of the proposed two universal portfolio strategies are used for the data analysis.

## Parameters of Finite Order Multinomial Universal Portfolio

Ten set of parameters $(p_1, p_2, N)$ are used for selection of good performances, they are (0.8, 0.001, 600), (0.001,0.8,600), (0.5, 0.5, 600), (0.9,0.1, 600), (0.1,0.9,600), (0.8,0.001,50), (0.001,0.8,50), (0.05,0.95,99), (0.95,0.05,99), (0.9,0.004, 10). In [6], parameter (0.8,0.001,600) was performed well when ran on three selected stocks from KLSE. Therefore, this parameters are chosen for the analysis. The other nine set of parameters are formed by permutation of (0.8,0.001,600).

## Parameters of Finite Order Multivariate Normal Universal Portfolio

In [7], good performances obtained by order one universal portfolio generated by Multivariate Normal Distribution with the set of parameters $(\mu_1, \mu_2, \mu_3, \sigma) = (7.1, 0.8, 0.2, 1.0)$. Therefore, order one of Multivariate Normal generated universal portfolio is studied and the ten sets of parameters are selected varying among (7.1,0.8,0.2,1.0), i.e. (7.1,0.8,0.2,1.0), (0.2,0.8,7.1,1.0), (0.8,7.1,0.2,1.0), (7.1,0.2,0.8,1.0), (0.8,0.2,7.1,1.0), (0.2,7.1,0.8,1.0), (8.1,9.1,0.1,2.0), (0.2,8.1,9.1,2.0), (8.1,0.2,9.1,2.0), (9,0.1,0.1,2.0).

**TABLE 2.** Average wealths obtained by Multinomial Generated Universal Portfolio better than CRP

| Strategies | Duration | Average Wealth | Average Standard Deviation |
|---|---|---|---|
| Multinomial [0.8, 0.001, 50] | Jan 2015 to Dec 2015 | 1.872272 | 0.144625964 |
| Multinomial [0.8, 0.001, 600] | Jan 2015 to Dec 2015 | 1.872129 | 0.1446316 |
| Multinomial [0.9, 0.004, 10] | Jan 2015 to Dec 2015 | 2.0260995 | 0.083495876 |
| Multinomial [0.9, 0.1, 600] | Jan 2015 to Dec 2015 | 2.021586 | 0.078432284 |
| Multinomial [0.95, 0.05, 99] | Jan 2015 to Dec 2015 | 2.105049 | 0.047998408 |
| Multinomial [0.8, 0.001, 50] | Jan 2014 to Dec 2015 | 6.24551 | 1.852848869 |
| Multinomial [0.8, 0.001, 600] | Jan 2014 to Dec 2015 | 6.2445335 | 1.853016454 |
| Multinomial [0.9, 0.004, 10] | Jan 2014 to Dec 2015 | 7.3133625 | 1.162690731 |
| Multinomial [0.9, 0.1, 600] | Jan 2014 to Dec 2015 | 7.3025065 | 1.205954352 |
| Multinomial [0.95, 0.05, 99] | Jan 2014 to Dec 2015 | 7.9577425 | 0.708231788 |
| Multinomial [0.9, 0.004, 10] | Jan 2013 to Dec 2015 | 30.06651151 | 8.336130634 |
| Multinomial [0.9, 0.1, 600] | Jan 2013 to Dec 2015 | 30.0945315 | 8.537531616 |
| Multinomial [0.95, 0.05, 99] | Jan 2013 to Dec 201 | 33.7422785 | 4.745774739 |

## RESULT AND DISCUSSION

The CRP wealth is a benchmark measurement for good performance. A comparison can be made with the wealth obtained by CRP and the wealths achieved by the universal portfolios generated by Multinomial distribution and Multivariate Normal distribution with the selected parametric vector stated in previous sections. Every one year inverval starting from year 2000 to year 2015 of the available stock data listed in Table 1 are used for study. At least 20 percent of the wealths achieved by proposed universal portfolio performed better than CRP are obtained for analysis. The good performance of the parameter is observed.

The average wealths and standard deviation obtained by Multinomial generated universal portfolio are shown in Tables 2, 3 and 4. Good performance is observed for selected parameters ( 0.9, 0.004, 10), ( 0.9, 0.1, 600) and ( 0.95, 0.05, 99) for period January 2004 to December 2015. The good performance parameter with higher average wealth 748.318 is (0.95,0.05,99).

The average wealth and standard deviation obtained by the Multivariate Normal generated universal portfolio are shown in Tables 5, 6 and 7. Wealth obtained higher than CRP is observed for selected parametric vectors ( 7.1,0.2,0.8,1.0), (7.1,0.8,0.2,1.0) and ( 9.0, 0.1, 0.1, 2.0) for period January 2004 to December 2015. The higher average wealth with the parameter (9.0,0.1,0.1,2.0).

**TABLE 3.** Average wealths obtained by Multinomial Generated Universal Portfolio better than CRP.

| Strategies | Duration | Average Wealth | Average Standard Deviation |
|---|---|---|---|
| Multinomial [0.8, 0.001, 50] | Jan 2008 to Dec 2015 | 158.612327 | 97.71612695 |
| Multinomial [0.8, 0.001, 600] | Jan 2008 to Dec 2015 | 158.587814 | 97.72132277 |
| Multinomial [0.9, 0.004, 10] | Jan 2008 to Dec 2015 | 237.065062 | 74.75428805 |
| Multinomial [0.9, 0.1, 600] | Jan 2008 to Dec 2015 | 238.6583255 | 77.68982841 |
| Multinomial [0.95, 0.05, 99] | Jan 2008 to Dec 2015 | 296.497761 | 48.91055484 |
| Multinomial [0.8, 0.001, 50] | Jan 2007 to Dec 2015 | 132.2665145 | 75.33927112 |
| Multinomial [0.8, 0.001, 600] | Jan 2007 to Dec 2015 | 132.2201705 | 75.31942405 |
| Multinomial [0.9, 0.004, 10] | Jan 2007 to Dec 2015 | 204.8146685 | 61.20016333 |
| Multinomial [0.9, 0.1, 600] | Jan 2007 to Dec 2015 | 204.964026 | 62.10088514 |
| Multinomial [0.95, 0.05, 99] | Jan 2007 to Dec 2015 | 258.1299895 | 41.37791389 |
| Multinomial [0.8, 0.001, 50] | Jan 2006 to Dec 2015 | 178.6115835 | 113.2260044 |
| Multinomial [0.8, 0.001, 600] | Jan 2006 to Dec 2015 | 178.537211 | 113.2070136 |
| Multinomial [0.9, 0.004, 10] | Jan 2006 to Dec 2015 | 277.5363855 | 92.07861702 |
| Multinomial [0.9, 0.1, 600] | Jan 2006 to Dec 2015 | 274.7625725 | 91.26704653 |
| Multinomial [0.95, 0.05, 99] | Jan 2006 to Dec 2015 | 347.7052635 | 58.89452853 |

**TABLE 4.** Average wealth obtained by Multinomial Generated Universal Portfolio better than CRP.

| Strategies | Duration | Average Wealth | Average Standard Deviation |
|---|---|---|---|
| Multinomial [0.8, 0.001, 50] | Jan 2005 to Dec 2015 | 239.105569 | 167.9310497 |
| Multinomial [0.8, 0.001, 600] | Jan 2005 to Dec 2015 | 239.00653 | 167.8962275 |
| Multinomial [0.9, 0.004, 10] | Jan 2005 to Dec 2015 | 380.6836375 | 141.8727569 |
| Multinomial [0.9, 0.1, 600] | Jan 2005 to Dec 2015 | 379.5841465 | 142.4877631 |
| Multinomial [0.95, 0.05, 99] | Jan 2005 to Dec 2015 | 487.8792055 | 95.32278617 |
| Multinomial [0.8, 0.001, 50] | Jan 2004 to Dec 2015 | 400.2781995 | 212.1060772 |
| Multinomial [0.8, 0.001, 600] | Jan 2004 to Dec 2015 | 400.128036 | 212.0530265 |
| Multinomial [0.9, 0.004, 10] | Jan 2004 to Dec 2015 | 604.2257875 | 157.7730287 |
| Multinomial [0.9, 0.1, 600] | Jan 2004 to Dec 2015 | 602.6234155 | 157.7730287 |
| Multinomial [0.95, 0.05, 99] | Jan 2004 to Dec 2015 | 748.3181105 | 102.8328358 |
| Multinomial [0.001, 0.8, 50] | Jan 2003 to Dec 2015 | 32.215131 | 11.89520483 |
| Multinomial [0.001, 0.8, 600] | Jan 2003 to Dec 2015 | 32.187138 | 11.91306069 |
| Multinomial [0.001, 0.8, 50] | Jan 2001 to Dec 2015 | 24.7398855 | 0.16061577 |
| Multinomial [0.001, 0.8, 600] | Jan 2001 to Dec 2015 | 24.730969 | 0.160571222 |

**TABLE 5.** Average wealths obtained by Multivariate Normal Generated Universal Portfolio better than CRP.

| Strategies | Duration | Average Wealth | Average Standard Deviation |
|---|---|---|---|
| MultivariateNormal [7.1, 0.2, 0.8, 1.0] | Jan 2015 to Dec 2015 | 1.955066 | 0.09180933 |
| Multivariate Normal [7.1, 0.8, 0.2, 1.0] | Jan 2015 to Dec 2015 | 1.956286 | 0.081379505 |
| Multivariate Normal [9.0, 0.1, 0.1, 2.0] | Jan 2015 to Dec 2015 | 2.0877165 | 0.041910926 |
| Multivariate Normal [7.1, 0.2, 0.8, 1.0] | Jan 2014 to Dec 2015 | 6.742861 | 1.501959857 |
| Multivariate Normal [7.1, 0.8, 0.2, 1.0] | Jan 2014 to Dec 2015 | 6.69516 | 1.395113194 |
| Multivariate Normal [9.0, 0.1, 0.1, 2.0] | Jan 2014 to Dec 2015 | 7.770994 | 0.763040342 |
| Multivariate Normal [7.1, 0.2, 0.8, 1.0] | Jan 2013 to Dec 2015 | 185.063506 | 72.81175258 |
| multivariate Normal [7.1, 0.8, 0.2, 1.0] | Jan 2013 to Dec 2015 | 193.523848 | 85.71626608 |
| Multivariate Normal[9.0, 0.1, 0.1, 2.0] | Jan 2013 to Dec 2015 | 276.3518625 | 51.95913896 |
| Multivariate Normal [7.1, 0.2, 0.8, 1.0] | Jan 2008 to Dec 2015 | 27.2618865 | 9.966086718 |
| multivariate Normal [7.1, 0.8, 0.2, 1.0] | Jan 2008 to Dec 2015 | 26.586557 | 8.681444109 |
| Multivariate Normal [9.0, 0.1, 0.1, 2.0] | Jan 2008 to Dec 2015 | 32.4116065 | 5.123314606 |

**TABLE 6.** Average wealths obtained by Multivariate Normal Generated Universal Portfolio better than CRP.

| Strategies | Duration | Average Wealth | Average Standard Deviation |
|---|---|---|---|
| Multivariate Normal [7.1, 0.2, 0.8, 1.0] | Jan 2007 to Dec 2015 | 164.975529 | 69.01928153 |
| multivariate Normal [7.1, 0.8, 0.2, 1.0] | Jan 2007 to Dec 2015 | 161.813746 | 64.6768412 |
| Multivariate Normal [9.0, 0.1, 0.1, 2.0] | Jan 2007 to Dec 2015 | 240.5946675 | 45.23218466 |
| Multivariate Normal [7.1, 0.2, 0.8, 1.0] | Jan 2006 to Dec 2015 | 225.052084 | 98.65878373 |
| multivariate Normal [7.1, 0.8, 0.2, 1.0] | Jan 2006 to Dec 2015 | 214.1665545 | 89.89713179 |
| Multivariate Normal [9.0, 0.1, 0.1, 2.0] | Jan 2006 to Dec 2015 | 327.6703685 | 65.94573513 |
| Multivariate Normal [7.1, 0.2, 0.8, 1.0] | Jan 2005 to Dec 2015 | 313.443567 | 154.3113924 |
| multivariate Normal [7.1, 0.8, 0.2, 1.0] | Jan 2005 to Dec 2015 | 310.560472 | 154.978443 |
| Multivariate Normal [9.0, 0.1, 0.1, 2.0] | Jan 2005 to Dec 2015 | 467.2951875 | 101.9507625 |

**TABLE 7.** Average wealths obtained by Multivariate Normal Generated Universal Portfolio better than CRP.

| Strategies | Duration | Average Wealth | Average Standard Deviation |
|---|---|---|---|
| Multivariate Normal [7.1, 0.2, 0.8, 1.0] | Jan 2004 to Dec 2015 | 490.154204 | 183.1714692 |
| multivariate Normal [7.1, 0.8, 0.2, 1.0] | Jan 2004 to Dec 2015 | 471.429849 | 159.3962751 |
| Multivariate Normal [9.0, 0.1, 0.1, 2.0] | Jan 2004 to Dec 2015 | 701.0537485 | 110.2752124 |
| Multivariate Normal [0.2, 0.8, 7.1, 1.0] | Jan 2003 to Dec 2015 | 44.052089 | 15.0078507 |
| multivariate Normal [0.8, 0.2, 7.1, 1.0] | Jan 2003 to Dec 2015 | 39.6614195 | 8.585701144 |
| Multivariate Normal [0.2, 8.1, 9.1, 2.0] | Jan 2001 to Dec 2015 | 25.8827735 | 1.301132339 |
| Multivariate Normal [0.2, 0.8, 7.1, 1.0] | Jan 2000 to Dec 2015 | 17.940328 | 7.129771817 |
| Multivariate Normal [0.2, 8.1, 9.1, 2.0] | Jan 2000 to Dec 2015 | 14.691917 | 4.566167495 |
| Multivariate Normal [0.8, 0.2, 7.1, 1.0] | Jan 2000 to Dec 2015 | 17.200072 | 6.209487897 |

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